

KEY

Section 1: Algebra

- 1.1 $3 - i, 3, 3 + i$
1.2 a, b
1.3 $H = \{\pm 1, \pm i\}; G/H \cong G$
1.4 6
1.5 c
1.6 Any two linearly independent vectors satisfying the conditions: $x - z + t = 0; 2x + 3y - z + 2t = 0$
1.7 a. 3; b. 1 or 2
1.8 $\lambda = 0$ with multiplicity $n - 1$ and $\lambda = n$ with multiplicity 1
1.9 $T^*(A) = M^*A$
1.10

$$\begin{bmatrix} \frac{1+\sqrt{3}}{2} & \frac{1-\sqrt{3}}{2} \\ \frac{1-\sqrt{3}}{2} & \frac{1+\sqrt{3}}{2} \end{bmatrix}$$

Section 2: Analysis

- 2.1 3
2.2 a, c
2.3 a, b
2.4 a, b, c
2.5 a, b, c
2.6 a
2.7 a, b, c
2.8 $6\pi iz_0$
2.9 a. $2\pi i$; b. π
2.10 a, b, c

Section 3: Topology

- 3.1 a
3.2 a, b, c
3.3 a, b, c
3.4 b
3.5 a. open; b. closed
3.6 a, b
3.7 b, c
3.8 b
3.9 a, b, c
3.10 b, c

Section 4: Calculus & Differential Equations

- 4.1 $\left(\frac{x}{\sqrt{x^2+16y^2}}, \frac{4y}{\sqrt{x^2+16y^2}}\right)$
4.2 $\sqrt{3}x + 2y - 4 = 0$
4.3 4π
4.4 $4\pi \int_0^a r^2 \varphi(r) dr$
4.5 $2ab$
4.6
$$\int_{-2}^1 \int_{-y}^{\sqrt{2-y}} f(x, y) dx dy + \int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x, y) dx dy$$

4.7 $\lambda = 0$ and $u = \text{constant}$; $\lambda = 4n^2\pi^2$ and $u_n = A \cos 2n\pi x + B \sin 2n\pi x$, for $n \in \mathbb{N}$
4.8 $\frac{1}{y^2} = cx^2 - x^4$
4.9
$$\begin{aligned} x(t) &= Ae^{3t} + Be^{2t} \\ y(t) &= Ae^{3t} + 2Be^{2t} \end{aligned}$$

4.10 $y = A \cos x + B \sin x$, or, equivalently, $y = c \sin(x - d)$.

Section 5: Miscellaneous

- 5.1 $\frac{\pi}{\sqrt{ab-h^2}}$
5.2 7π
5.3 b, c
5.4 $\frac{1}{2}N\phi(N)$
5.5 a, b, c
5.6 a, b, c
5.7 b, c
5.8 46
5.9 $8! \binom{9}{6} = 8! \binom{9}{3} = 3,386,880$
5.10
$$r \binom{n}{r} = n \binom{n-1}{r-1}$$

Note: Accept any correct equivalent form of the answers.