

**KEY****Section 1: Algebra**

- 1.1 a,b,c  
 1.2 a,b,c  
 1.3 a,c  
 1.4  $\frac{p-1}{2}$   
 1.5 3 or 5  
 1.6 2, 3, 4  
 1.7  $A^{-1} = \frac{4}{3}I - \frac{1}{3}A$   
 1.8 a  
 1.9 c  
 1.10  $W^\perp = \{aI \mid a \in \mathbb{R}\}$

**Section 2: Analysis**

- 2.1 a,c  
 2.2 b  
 2.3 a,b,c  
 2.4 a,b,c  
 2.5 b,c  
 2.6 Only  $x = 1$  and  $f'(1) = 2$   
 2.7 a,c  
 2.8 c  
 2.9

$$\sum_{n=1}^{\infty} \frac{1}{z^n} + \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$$

- 2.10 a,b

**Section 3: Topology**

- 3.1 a  
 3.2 None  
 3.3 a,b  
 3.4 a,b  
 3.5 b,c  
 3.6 a,b  
 3.7 a,b,c  
 3.8 None  
 3.9 a,b,c  
 3.10 a,b,c

**Section 4: Calculus & Differential Equations**

4.1

$$\int_{x^2}^{x^3} y^2 \sec^2(xy^2) dy + 3x^2 \tan x^7 - 2x \tan x^5$$

4.2

$$\sqrt{\frac{\pi}{2}}$$

4.3 54 $\pi$ .

4.4  $2\pi$

4.5

$$\int_{\Omega} (v\Delta u - u\Delta v) dx dy dz,$$

where  $\Delta$  is the Laplace operator.

4.6  $\frac{1+\sqrt{2}}{2}$

4.7

$$\int_{-2}^1 \int_{-y}^{\sqrt{2-y}} f(x,y) dx dy + \int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x,y) dx dy$$

4.8

$$\lambda_n = \frac{(2n-1)^2 \pi^2}{4}, u_n = C \sin \frac{(2n-1)\pi x}{2}, n \in \mathbb{N}$$

4.9  $-2 < a < -1$

4.10

$$u(x,1) = \begin{cases} 1, & \text{if } |x| < 1, \\ \frac{1}{2}, & \text{if } 1 < |x| < 3, \\ 0, & \text{if } |x| > 3. \end{cases}$$

**Section 5: Miscellaneous**

5.1  $2^{\frac{n}{2}} \sin(x + \frac{n\pi}{4})$

5.2 b

5.3 a,c

5.4  $\frac{2\pi}{3}$

5.5  $\frac{n}{2} \sin \frac{2\pi}{n}$

5.6 19

5.7 a.  $\frac{5}{11}$ , b.  $\frac{53}{66}$

5.8  $\frac{3}{4} - \log 2$

5.9 12

5.10

$$\prod_{1 \leq i < j \leq 4} (a_i - a_j).$$

**Note:** Please accept any correct equivalent form of the answers.