Q.1

Items	Cost (₹)	Profit %	Marked Price (₹)
P	5,400	---	5,860
Q	---	25	10,000

Details of prices of two items P and Q are presented in the above table. The ratio of cost of item P to cost of item Q is 3:4. Discount is calculated as the difference between the marked price and the selling price. The profit percentage is calculated as the ratio of the difference between selling price and cost, to the cost (Profit % $= \frac{\text{Selling price} - \text{Cost}}{\text{Cost}} \times 100$).

The discount on item Q, as a percentage of its marked price, is ______

Options
1. 25
2. 10
3. 12.5
4. 5

Question Type : **MCQ**
Question ID : **8232511177**
Status : **Answered**
Chosen Option : **2**

Q.2 Given below are two statements 1 and 2, and two conclusions I and II.

Statement 1: All bacteria are microorganisms.

Statement 2: All pathogens are microorganisms.

Conclusion I: Some pathogens are bacteria.

Conclusion II: All pathogens are not bacteria.

Based on the above statements and conclusions, which one of the following options is logically CORRECT?

Options

1. Either conclusion I or II is correct.
2. Only conclusion II is correct
3. Neither conclusion I nor II is correct.
4. Only conclusion I is correct

Question Type : **MCQ**
Question ID : **8232511179**
Status : **Answered**
Chosen Option : **3**

Q.3 A polygon is convex if, for every pair of points, P and Q belonging to the polygon, the line segment PQ lies completely inside or on the polygon.

Which one of the following is <u>NOT</u> a convex polygon?

Options

1. 

2. 

3. 

4. 

Question Type : **MCQ**
Question ID : **8232511172**
Status : **Answered**
Chosen Option : **1**

Q.4 There are five bags each containing identical sets of ten distinct chocolates. One chocolate is picked from each bag.

The probability that at least two chocolates are identical is ___________

Options
1. 0.6976
2. 0.4235
3. 0.3024
4. 0.8125

Question Type : **MCQ**
Question ID : **8232511178**
Status : **Answered**
Chosen Option : **1**

Q.5 _____ is to *surgery* as *writer* is to ________

Which one of the following options maintains a similar logical relation in the above sentence?

Options
1. Doctor, book
2. Hospital, library
3. Plan, outline
4. Medicine, grammar

Question Type : **MCQ**
Question ID : **8232511175**
Status : **Marked For Review**
Chosen Option : **1**

Q.6

Some people suggest anti-obesity measures (AOM) such as displaying calorie information in restaurant menus. Such measures sidestep addressing the core problems that cause obesity: poverty and income inequality.

Which one of the following statements summarizes the passage?

Options

1. AOM are addressing the problem superficially.
2. AOM are addressing the core problems and are likely to succeed.
3. If obesity reduces, poverty will naturally reduce, since obesity causes poverty.
4. The proposed AOM addresses the core problems that cause obesity.

Question Type :	**MCQ**
Question ID :	**8232511180**
Status :	**Not Attempted and Marked For Review**
Chosen Option :	**--**

Q.7 We have 2 rectangular sheets of paper, M and N, of dimensions 6 cm x 1 cm each. Sheet M is rolled to form an open cylinder by bringing the short edges of the sheet together. Sheet N is cut into equal square patches and assembled to form the largest possible closed cube. Assuming the ends of the cylinder are closed, the ratio of the volume of the cylinder to that of the cube is ___________

Options

1. $\frac{3}{\pi}$
2. $\frac{\pi}{2}$
3. $\frac{9}{\pi}$
4. 3π

Question Type : **MCQ**
Question ID : **8232511176**
Status : **Answered**
Chosen Option : **3**

Q.8 Consider the following sentences:

(i) Everybody in the class is prepared for the exam.

(ii) Babu invited Danish to his home because he enjoys playing chess.

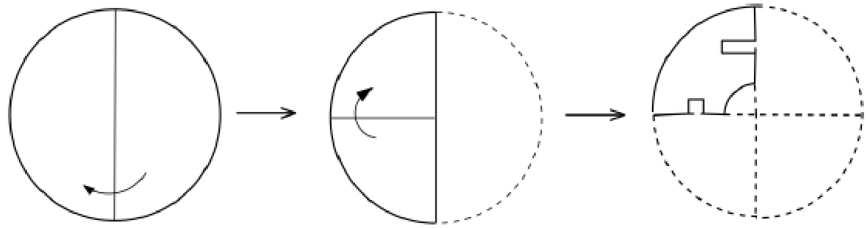
Which of the following is the CORRECT observation about the above two sentences?

Options

1. (i) is grammatically incorrect and (ii) is ambiguous
2. (i) is grammatically correct and (ii) is ambiguous
3. (i) is grammatically incorrect and (ii) is unambiguous
4. (i) is grammatically correct and (ii) is unambiguous

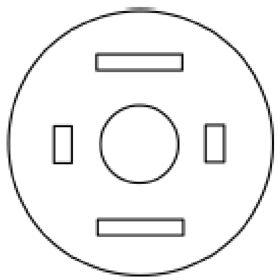
Question Type : **MCQ**
Question ID : **8232511173**
Status : **Answered**
Chosen Option : **3**

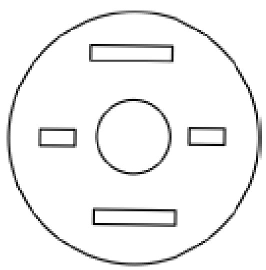
Q.9

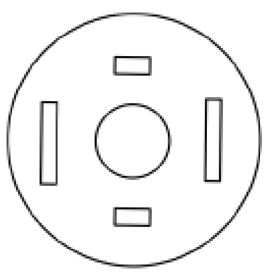


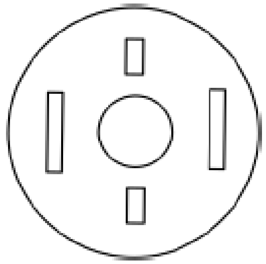
A circular sheet of paper is folded along the lines in the directions shown. The paper, after being punched in the final folded state as shown and unfolded in the reverse order of folding, will look like _______.

Options

1. 

2. 

3. 

4. 

Question Type : **MCQ**
Question ID : **8232511174**
Status : **Answered**
Chosen Option : **1**

Q.10 The ratio of boys to girls in a class is 7 to 3.

Among the options below, an acceptable value for the total number of students in the class is:

Options
1. 21
2. 50
3. 37
4. 73

Question Type : **MCQ**
Question ID : **8232511171**
Status : **Answered**
Chosen Option : **1**

Section : **MA Mathematics**

Q.1 Let $L^2[-1,1]$ be the Hilbert space of real valued square integrable functions on $[-1,1]$ equipped with the norm $\|f\| = \left(\int_{-1}^{1} |f(x)|^2\, dx\right)^{1/2}$.

Consider the subspace $M = \{f \in L^2[-1,1] \,:\, \int_{-1}^{1} f(x)dx = 0\}$.

For $f(x) = x^2$, define $d = \inf\{\|f - g\| : g \in M\}$. Then

Options

1. $d = \dfrac{3}{\sqrt{2}}$
2. $d = \dfrac{3}{2}$
3. $d = \dfrac{\sqrt{2}}{3}$
4. $d = \dfrac{2}{3}$

Question Type : **MCQ**
Question ID : **8232511213**
Status : **Not Answered**
Chosen Option : **--**

Q.2 Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a twice continuously differentiable scalar field such that $div(\nabla f) = 6$. Let S be the surface $x^2 + y^2 + z^2 = 1$ and $\hat{n}$ be unit outward normal to S. Then the value of $\iint_S (\nabla f \cdot \hat{n})\ dS$ is

Options

1. $6\,\pi$
2. $2\,\pi$
3. $8\,\pi$
4. $4\,\pi$

Question Type : **MCQ**
Question ID : **8232511192**
Status : **Answered**
Chosen Option : **3**

Q.3 Let $y(t)$ be the solution of the initial value problem

$$\frac{d^2y}{dt^2} + a\frac{dy}{dt} + b\,y = f(t), \qquad a > 0, \qquad b > 0, \qquad a \neq b, \qquad a^2 - 4b = 0,$$

$$y(0) = 0, \quad \frac{dy}{dt}(0) = 0,$$

obtained by the method of Laplace transform. Then

Options

1. $y(t) = \int_0^t e^{\frac{-a\tau}{2}} f(t-\tau)\, d\tau$
2. $y(t) = \int_0^t \tau\, e^{\frac{-a\tau}{2}} f(t-\tau)\, d\tau$
3. $y(t) = \int_0^t \tau\, e^{\frac{-b\tau}{2}} f(t-\tau)\, d\tau$
4. $y(t) = \int_0^t e^{\frac{-b\tau}{2}} f(t-\tau)\, d\tau$

Question Type : **MCQ**
Question ID : **8232511209**
Status : **Not Answered**
Chosen Option : **--**

Q.4 Let H be a complex Hilbert space. Let $u, v \in H$ be such that $\langle u, v\rangle = 2$. Then

$$\frac{1}{2\pi}\int_0^{2\pi} \left\|u + e^{it}v\right\|^2 e^{it}\, dt = \_\_\_\_\_\_\_\_\_.$$

Given Answer : **--**

Question Type : **NAT**
Question ID : **8232511205**
Status : **Not Answered**

Q.5 If the polynomial

$$p(x) = \alpha + \beta(x+2) + \gamma(x+2)(x+1) + \delta(x+2)(x+1)x$$

interpolates the data

x	-2	-1	0	1	2
$f(x)$	2	-1	8	5	-34

then $\alpha + \beta + \gamma + \delta =$ ___________ .

Given Answer : **1**

Question Type : **NAT**
Question ID : **8232511202**
Status : **Answered**

Q.6 Let $f(z) = u(x,y) + i\,v(x,y)$ for $z = x + iy \in \mathbb{C}$, where x and y are real numbers, be a non-constant analytic function on the complex plane $\mathbb{C}$. Let u_x, v_x and u_y, v_y denote the first order partial derivatives of $u(x,y) = Re(f(z))$ and $v(x,y) = Im(f(z))$ with respect to real variables x and y, respectively. Consider the following two functions defined on $\mathbb{C}$:

$$g_1(z) = u_x(x,y) - i\,u_y\,(x,y) \text{ for } z = x + iy \in \mathbb{C},$$

$$g_2(z) = v_x(x,y) + i\,v_y(x,y) \text{ for } z = x + iy \in \mathbb{C}.$$

Then

Options

1. neither $g_1(z)$ nor $g_2(z)$ is analytic in $\mathbb{C}$
2. $g_1(z)$ is NOT analytic in $\mathbb{C}$ and $g_2(z)$ is analytic in $\mathbb{C}$
3. $g_1(z)$ is analytic in $\mathbb{C}$ and $g_2(z)$ is NOT analytic in $\mathbb{C}$
4. both $g_1(z)$ and $g_2(z)$ are analytic in $\mathbb{C}$

Question Type : **MCQ**
Question ID : **8232511182**
Status : **Answered**
Chosen Option : **3**

Q.7 Let $D = \{z \in \mathbb{C} \;:\; |z| < 2\pi\}$ and $f: D \to \mathbb{C}$ be the function defined by

$$f(z) = \begin{cases} \dfrac{3\,z^2}{(1 - \cos z)} & \text{if } z \neq 0, \\ 6 & \text{if } z = 0. \end{cases}$$

If $f(z) = \sum_{n=0}^{\infty} a_n\, z^n$ for $z \in D$, then $6\,a_2 =$ __________.

Given Answer : **0.75**

Question Type : **NAT**
Question ID : **8232511227**
Status : **Marked For Review**

Q.8 Let

$$f(x) = x^4 + 2x^3 - 11x^2 - 12x + 36 \quad \text{for } x \in \mathbb{R}.$$

The order of convergence of the Newton-Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n \geq 0,$$

with $x_0 = 2.1$, for finding the root $\alpha = 2$ of the equation $f(x) = 0$ is _______ .

Given Answer : **--**

Question Type :	**NAT**
Question ID :	**8232511201**
Status :	**Not Answered**

Q.9 If $u(x,t) = A\, e^{-t} \sin x$ solves the following initial boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \qquad t > 0,$$

$$u(0,t) = u(\pi,t) = 0, \qquad t > 0,$$

$$u(x,0) = \begin{cases} 60, & 0 < x \leq \frac{\pi}{2}, \\ 40, & \frac{\pi}{2} < x < \pi, \end{cases}$$

then $\pi A =$ _______ .

Given Answer : **188.4**

Question Type :	**NAT**
Question ID :	**8232511231**
Status :	**Answered**

Q.10 The eigenvalues of the boundary value problem

$$\frac{d^2y}{dx^2} + \lambda y = 0, \qquad x \in (0, \pi), \ \ \lambda > 0,$$

$$y(0) = 0, \qquad y(\pi) - \frac{dy}{dx}(\pi) = 0,$$

are given by

Options 1.

$\lambda = k_n^2$, where $k_n, n = 1,2,3, \dots$ are the roots of $k - \tan(k\pi) = 0$

2. $\lambda = (n\pi)^2, \qquad n = 1,2,3, \dots$

3. $\lambda = n^2, \qquad n = 1,2,3, \dots$

4.

$\lambda = k_n^2$, where $k_n, n = 1,2,3, \dots$ are the roots of $k + \tan(k\pi) = 0$

Question Type : **MCQ**

Question ID : **8232511185**

Status : **Answered**

Chosen Option : **1**

Q.11 Consider the following topologies on the set $\mathbb{R}$ of all real numbers.

$\mathcal{T}_1$ is the upper limit topology having all sets $(a, b]$ as basis.

$\mathcal{T}_2 = \{U \subset \mathbb{R} : \mathbb{R} \backslash U \text{ is finite}\} \cup \{\emptyset\}$.

$\mathcal{T}_3$ is the standard topology having all sets (a, b) as basis.

Then

Options
1. $\mathcal{T}_3 \subset \mathcal{T}_2 \subset \mathcal{T}_1$
2. $\mathcal{T}_1 \subset \mathcal{T}_2 \subset \mathcal{T}_3$
3. $\mathcal{T}_2 \subset \mathcal{T}_1 \subset \mathcal{T}_3$
4. $\mathcal{T}_2 \subset \mathcal{T}_3 \subset \mathcal{T}_1$

Question Type : **MCQ**
Question ID : **8232511221**
Status : **Answered**
Chosen Option : **3**

Q.12 Consider the following statements:

P: $d_1(x, y) = \left|\log\left(\frac{x}{y}\right)\right|$ is a metric on $(0, 1)$.

Q: $d_2(x, y) = \begin{cases} |x| + |y|, & \text{if } x \neq y, \\ 0, & \text{if } x = y, \end{cases}$ is a metric on $(0, 1)$.

Then

Options

1. both P and Q are TRUE
2. P is FALSE and Q is TRUE
3. both P and Q are FALSE
4. P is TRUE and Q is FALSE

Question Type : **MCQ**
Question ID : **8232511191**
Status : **Marked For Review**
Chosen Option : **1**

Q.13 Let $\langle\cdot,\ \cdot\rangle: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be an inner product on the vector space $\mathbb{R}^n$ over $\mathbb{R}$.

Consider the following statements:

P: $|\langle u, v\rangle| \leq \frac{1}{2}(\langle u, u\rangle + \langle v, v\rangle)$ for all $u, v \in \mathbb{R}^n$.

Q: If $\langle u, v\rangle = \langle 2u, -v\rangle$ for all $v \in \mathbb{R}^n$, then $u = 0$.

Then

Options
1. both P and Q are FALSE
2. P is TRUE and Q is FALSE
3. P is FALSE and Q is TRUE
4. both P and Q are TRUE

Question Type : **MCQ**
Question ID : **8232511223**
Status : **Answered**
Chosen Option : **4**

Q.14 Let G be a group of order 5^4 with center having 5^2 elements. Then the number of conjugacy classes in G is __________ .

Given Answer : **--**

Question Type : **NAT**
Question ID : **8232511224**
Status : **Not Answered**

Q.15 Let $R = \{z = x + iy \in \mathbb{C} \ : \ 0 < x < 1 \text{ and } -11\,\pi < y < 11\,\pi\}$ and Γ be the positively oriented boundary of R. Then the value of the integral

$$\frac{1}{2\pi i}\int_{\Gamma}\frac{e^{z}\ dz}{e^{z}-2}$$

is ________ .

Given Answer : **0**

Question Type : **NAT**
Question ID : **8232511226**
Status : **Answered**

Q.16 Let $f_n : [0, 10] \to \mathbb{R}$ be given by $f_n(x) = n\,x^3\,e^{-nx}$ for $n = 1,2,3, \dots$.

Consider the following statements:

P: (f_n) is equicontinuous on $[0, 10]$.

Q: $\sum_{n=1}^{\infty} f_n$ does NOT converge uniformly on $[0, 10]$.

Then

Options
1. both P and Q are FALSE
2. both P and Q are TRUE
3. P is FALSE and Q is TRUE
4. P is TRUE and Q is FALSE

Question Type : **MCQ**
Question ID : **8232511216**
Status : **Marked For Review**
Chosen Option : **4**

Q.17 Let $f: \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by $f(x) = \frac{\pi}{2} + x - \tan^{-1} x$. Consider the following statements:

P: $|f(x) - f(y)| < |x - y|$ for all $x, y \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Q: f has a fixed point.

Then

Options
1. P is TRUE and Q is FALSE
2. both P and Q are TRUE
3. both P and Q are FALSE
4. P is FALSE and Q is TRUE

Question Type : **MCQ**
Question ID : **8232511190**
Status : **Answered**
Chosen Option : **1**

Q.18 The initial value problem

$$\frac{dy}{dt} = f(t,y), \quad t > 0, \quad y(0) = 1,$$

where $f(t,y) = -10\,y$, is solved by the following Euler method

$$y_{n+1} = y_n + h\,f(t_n, y_n), \quad n \geq 0,$$

with step-size h. Then $y_n \to 0$ as $n \to \infty$, provided

Options
1. $0.5 < h < 0.55$
2. $0.3 < h < 0.4$
3. $0 < h < 0.2$
4. $0.4 < h < 0.5$

Question Type : **MCQ**
Question ID : **8232511211**
Status : **Not Answered**
Chosen Option : **--**

Q.19 For each $x \in (0,1]$, consider the decimal representation $x = \cdot\, d_1 d_2 d_3 \cdots d_i \cdots$.
Define $f: [0,1] \to \mathbb{R}$ by $f(x) = 0$ if x is rational and $f(x) = 18\,n$ if x is irrational, where n is the number of zeroes immediately after the decimal point up to the first nonzero digit in the decimal representation of x. Then the Lebesgue integral

$\int_0^1 f(x)\,dx =$ ________.

Given Answer : **--**

Question Type : **NAT**
Question ID : **8232511234**
Status : **Not Answered**

Q.20 The family of surfaces given by $u = xy + f(x^2 - y^2)$, where $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function, satisfies

Options

1. $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = x^2 + y^2$
2. $y\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial y} = x^2 - y^2$
3. $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = x^2 - y^2$
4. $y\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial y} = x^2 + y^2$

Question Type : **MCQ**
Question ID : **8232511186**
Status : **Answered**
Chosen Option : **4**

Q.21 Let R be the row reduced echelon form of a 4×4 real matrix A and let the third column of R be $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. Consider the following statements:

P: If $\begin{bmatrix} \alpha \\ \beta \\ \gamma \\ 0 \end{bmatrix}$ is a solution of $A\mathbf{x} = \mathbf{0}$, then $\gamma = 0$.

Q: For all $\mathbf{b} \in \mathbb{R}^4$, $rank[A|\,\mathbf{b}] = rank[R|\,\mathbf{b}]$.

Then

Options
1. P is FALSE and Q is TRUE
2. P is TRUE and Q is FALSE
3. both P and Q are TRUE
4. both P and Q are FALSE

Question Type : **MCQ**
Question ID : **8232511184**
Status : **Answered**
Chosen Option : **1**

Q.22 Consider the fixed-point iteration

$$x_{n+1} = \varphi(x_n), \; n \geq 0,$$

with $\varphi(x) = 3 + (x-3)^3, \; x \in (2.5, \; 3.5),$

and the initial approximation $x_0 = 3.25.$

Then, the order of convergence of the fixed-point iteration method is

Options
1. 4
2. 2
3. 1
4. 3

Question Type : **MCQ**
Question ID : **8232511188**
Status : **Not Answered**
Chosen Option : **--**

Q.23 Let $\mathbb{Z}$ denote the ring of integers. Consider the subring

$R = \{a + b\sqrt{-17} : \ a, b \in \mathbb{Z}\}$ of the field $\mathbb{C}$ of complex numbers.

Consider the following statements:

P: $2 + \sqrt{-17}$ is an irreducible element.

Q: $2 + \sqrt{-17}$ is a prime element.

Then

Options
1. both P and Q are FALSE
2. P is TRUE and Q is FALSE
3. P is FALSE and Q is TRUE
4. both P and Q are TRUE

Question Type : **MCQ**
Question ID : **8232511206**
Status : **Answered**
Chosen Option : **1**

Q.24 If $y = \sum_{k=0}^{\infty} a_k x^k$, $(a_0 \neq 0)$ is the power series solution of the differential equation $\frac{d^2y}{dx^2} - 24\,x^2 y = 0$, then $\frac{a_4}{a_0} =$ __________.

Given Answer : **2**

Question Type : **NAT**
Question ID : **8232511230**
Status : **Answered**

Q.25 Let $\{e_n : n = 1,2,3,\dots\}$ be an orthonormal basis of a complex Hilbert space H.

Consider the following statements:

P: There exists a bounded linear functional $f: H \to \mathbb{C}$ such that $f(e_n) = \frac{1}{n}$ for $n = 1,2,3,\dots$.

Q: There exists a bounded linear functional $g: H \to \mathbb{C}$ such that $g(e_n) = \frac{1}{\sqrt{n}}$ for $n = 1,2,3,\dots$.

Then

Options

1. both P and Q are TRUE
2. P is FALSE and Q is TRUE
3. P is TRUE and Q is FALSE
4. both P and Q are FALSE

Question Type : **MCQ**
Question ID : **8232511189**
Status : **Not Answered**
Chosen Option : **--**

Q.26 The equation $xy - z\log y + e^{xz} = 1$ can be solved in a neighborhood of the point $(0,1,1)$ as $y = f(x,z)$ for some continuously differentiable function f.

Then

Options

1. $\nabla f(0,1) = (0,2)$
2. $\nabla f(0,1) = (1,0)$
3. $\nabla f(0,1) = (0,1)$
4. $\nabla f(0,1) = (2,0)$

Question Type : **MCQ**
Question ID : **8232511220**
Status : **Answered**
Chosen Option : **2**

Q.27 Let A be a 3×4 matrix and B be a 4×3 matrix with real entries such that AB is non-singular. Consider the following statements:

P: Nullity of A is 0.

Q: BA is a non-singular matrix.

Then

Options
1. both P and Q are TRUE
2. P is TRUE and Q is FALSE
3. both P and Q are FALSE
4. P is FALSE and Q is TRUE

Question Type : **MCQ**
Question ID : **8232511181**
Status : **Answered**
Chosen Option : **2**

Q.28 If the quadrature formula

$$\int_0^2 x\, f(x)\, dx \approx \alpha\, f(0) + \beta\, f(1) + \gamma\, f(2)$$

is exact for all polynomials of degree ≤ 2, then $2\beta - \gamma =$ _______.

Given Answer : **2**

Question Type : **NAT**
Question ID : **8232511233**
Status : **Answered**

Q.29 Let I be the ideal generated by $x^2 + x + 1$ in the polynomial ring $R = \mathbb{Z}_3[x]$, where $\mathbb{Z}_3$ denotes the ring of integers modulo 3. Then the number of units in the quotient ring R/I is ________ .

Given Answer : --

Question Type : **NAT**
Question ID : **8232511198**
Status : **Not Attempted and Marked For Review**

Q.30 Consider the Linear Programming Problem P:

$$\text{Minimize } 2x_1 - 5x_2$$

subject to

$$2x_1 + 3x_2 + s_1 = 12,$$

$$-x_1 + x_2 + s_2 = 1,$$

$$-x_1 + 2x_2 + s_3 = 3,$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad s_1 \geq 0, \quad s_2 \geq 0, \text{ and } s_3 \geq 0.$$

If $\begin{bmatrix} x_1 \\ 2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}$ is a basic feasible solution of P, then $x_1 + s_1 + s_2 + s_3 =$ ________ .

Given Answer : --

Question Type : **NAT**
Question ID : **8232511204**
Status : **Not Answered**

Q.31 Consider the second-order partial differential equation (PDE)

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + (x^2 + 4y^2)\frac{\partial^2 u}{\partial y^2} = \sin(x + y).$$

Consider the following statements:

P: The PDE is parabolic on the ellipse $\frac{x^2}{4} + y^2 = 1$.

Q: The PDE is hyperbolic inside the ellipse $\frac{x^2}{4} + y^2 = 1$.

Then

Options
1. both P and Q are TRUE
2. P is TRUE and Q is FALSE
3. P is FALSE and Q is TRUE
4. both P and Q are FALSE

Question Type : **MCQ**
Question ID : **8232511207**
Status : **Answered**
Chosen Option : **1**

Q.32 The number of zeroes (counting multiplicity) of $P(z) = 3z^5 + 2i\, z^2 + 7i\, z + 1$ in the annular region $\{z \in \mathbb{C} \ : 1 < \ |z| < 7\}$ is ________ .

Given Answer : **5**

Question Type : **NAT**
Question ID : **8232511228**
Status : **Answered**

Q.33 Let V be the solid region in $\mathbb{R}^3$ bounded by the paraboloid $y = (x^2 + z^2)$ and the plane $y = 4$. Then the value of $\iiint_V 15\sqrt{x^2 + z^2}\, dV$ is

Options
1. 256π
2. 28π
3. 128π
4. 64π

Question Type : **MCQ**
Question ID : **8232511218**
Status : **Answered**
Chosen Option : **3**

Q.34 Consider the Linear Programming Problem P:

$$\text{Maximize } c_1x_1 + c_2x_2$$

subject to

$$a_{11}x_1 + a_{12}x_2 \le b_1,$$

$$a_{21}x_1 + a_{22}x_2 \le b_2,$$

$$a_{31}x_1 + a_{32}x_2 \le b_3,$$

$x_1 \ge 0$ and $x_2 \ge 0$, where a_{ij}, b_i and c_j are real numbers ($i = 1,2,3; j = 1,2$).

Let $\begin{bmatrix} p \\ q \end{bmatrix}$ be a feasible solution of P such that $p\,c_1 + q\,c_2 = 6$ and let all feasible solutions $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ of P satisfy $-5 \le c_1x_1 + c_2x_2 \le 12$.

Then, which one of the following statements is NOT true?

Options
1. The dual of P has at least one feasible solution
2. If $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ is a feasible solution of the dual of P, then $b_1y_1 + b_2y_2 + b_3y_3 \ge 6$
3. The feasible region of P is a bounded set
4. P has an optimal solution

Question Type : **MCQ**
Question ID : **8232511212**
Status : **Not Answered**
Chosen Option : **--**

Q.35 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be differentiable. Let $D_u f(0,0)$ and $D_v f(0,0)$ be the directional derivatives of f at $(0,0)$ in the directions of the unit vectors $u = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ and $v = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$, respectively. If $D_u f(0,0) = \sqrt{5}$ and $D_v f(0,0) = \sqrt{2}$, then $\frac{\partial f}{\partial x}(0,0) + \frac{\partial f}{\partial y}(0,0) =$ ______ .

Given Answer : **4**

Question Type : **NAT**
Question ID : **8232511195**
Status : **Answered**

Q.36 Let Γ denote the boundary of the square region R with vertices $(0,0)$, $(2,0)$, $(2,2)$ and $(0,2)$ oriented in the counter-clockwise direction. Then

$$\oint_{\Gamma} (1 - y^2)\, dx + x\, dy = \underline{\qquad\qquad}.$$

Given Answer : **12**

Question Type : **NAT**
Question ID : **8232511196**
Status : **Answered**

Q.37 The function $u(x,t)$ satisfies the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad x \in \mathbb{R}, \quad t > 0,$$

$$u(x,0) = 0, \quad \frac{\partial u}{\partial t}(x,0) = 4xe^{-x^2}.$$

Then $u(5,5)$ is

Options

1. $1 - e^{100}$
2. $1 - \frac{1}{e^{100}}$
3. $1 - \frac{1}{e^{10}}$
4. $1 - e^{10}$

Question Type : **MCQ**
Question ID : **8232511187**
Status : **Answered**
Chosen Option : **2**

Q.38 Let $V = \{p : p(x) = a_0 + a_1x + a_2x^2, \ a_0, a_1, a_2 \in \mathbb{R}\}$ be the vector space of all polynomials of degree at most 2 over the real field $\mathbb{R}$. Let $T: V \to V$ be the linear operator given by

$$T(p) = (p(0) - p(1)) + (p(0) + p(1))\,x + p(0)\,x^2.$$

Then the sum of the eigenvalues of T is _____ .

Given Answer : **1**

Question Type : **NAT**
Question ID : **8232511232**
Status : **Marked For Review**

Q.39 Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-1\\1\end{bmatrix}, \; T^2\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix}, \text{ and } T^2\left(\begin{bmatrix}1\\1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix}.$$

Then the rank of T is ______ .

Given Answer : **2**

Question Type : **NAT**
Question ID : **8232511199**
Status : **Marked For Review**

Q.40 The number of 5-Sylow subgroups in the symmetric group S_5 of degree 5 is ___ .

Given Answer : **6**

Question Type : **NAT**
Question ID : **8232511197**
Status : **Marked For Review**

Q.41 Let F be a finite field and $F^{\times}$ be the group of all nonzero elements of F under multiplication. If $F^{\times}$ has a subgroup of order 17, then the smallest possible order of the field F is ________ .

Given Answer : **--**

Question Type : **NAT**
Question ID : **8232511225**
Status : **Not Attempted and Marked For Review**

Q.42 Let $y(x)$ be the solution of the following initial value problem

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0, \qquad x > 0,$$

$$y(2) = 0, \quad \frac{dy}{dx}(2) = 4.$$

Then $y(4) =$ ___________.

Given Answer : **32**

Question Type : **NAT**
Question ID : **8232511200**
Status : **Answered**

Q.43 Consider the Linear Programming Problem P:

$$\text{Maximize } 2x_1 + 3x_2$$

subject to

$$2x_1 + x_2 \le 6,$$

$$-x_1 + x_2 \le 1,$$

$$x_1 + x_2 \le 3,$$

$$x_1 \ge 0 \text{ and } x_2 \ge 0.$$

Then the optimal value of the dual of P is equal to ________ .

Given Answer : **8**

Question Type : **NAT**
Question ID : **8232511203**
Status : **Answered**

Q.44 Let $\ell^1 = \{x = (x(1), x(2), \dots, x(n), \dots) \mid \sum_{n=1}^{\infty} |x(n)| < \infty\}$ be the sequence space equipped with the norm $\|x\| = \sum_{n=1}^{\infty} |x(n)|$. Consider the subspace

$$X = \left\{x \in \ell^1 : \sum_{n=1}^{\infty} n\,|x(n)| < \infty\right\},$$

and the linear transformation $T: X \to \ell^1$ given by

$(Tx)(n) = n\,x(n)$ for $n = 1,2,3,\dots$. Then

Options
1. T is closed but NOT bounded
2. T is neither closed nor bounded
3. T is bounded
4. T^{-1} exists and is an open map

Question Type : **MCQ**
Question ID : **8232511215**
Status : **Not Answered**
Chosen Option : **--**

Q.45 If $u(x, y)$ is the solution of the Cauchy problem

$$x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1, \qquad u(x,0) = -x^2, \quad x > 0,$$

then $u(2,1)$ is equal to

Options
1. $1 + 2\,e^{-2}$
2. $1 + 4\,e^{-2}$
3. $1 - 2\,e^{-2}$
4. $1 - 4\,e^{-2}$

Question Type : **MCQ**
Question ID : **8232511208**
Status : **Answered**
Chosen Option : **3**

Q.46 Consider the following statements:

P: Every compact metrizable topological space is separable.

Q: Every Hausdorff topology on a finite set is metrizable.

Then

Options
1. both P and Q are TRUE
2. P is FALSE and Q is TRUE
3. both P and Q are FALSE
4. P is TRUE and Q is FALSE

Question Type : **MCQ**
Question ID : **8232511193**
Status : **Not Answered**
Chosen Option : **--**

Q.47 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) = 4xy - 2x^2 - y^4$. Then f has

Options
1. a point of local maximum and a saddle point
2. a point of local maximum and a point of local minimum
3. a point of local minimum and a saddle point
4. two saddle points

Question Type : **MCQ**
Question ID : **8232511219**
Status : **Answered**
Chosen Option : **1**

Q.48 Let A be a square matrix such that $\det(xI - A) = x^4(x-1)^2(x-2)^3$, where $\det(M)$ denotes the determinant of a square matrix M.

If $\operatorname{rank}(A^2) < \operatorname{rank}(A^3) = \operatorname{rank}(A^4)$, then the geometric multiplicity of the eigenvalue 0 of A is ________ .

Given Answer : --

Question Type : **NAT**
Question ID : **8232511229**
Status : **Not Answered**

Q.49 Let $\mathbb{R}$ denote the set of all real numbers. Consider the following topological spaces.

$X_1 = (\mathbb{R}, \mathcal{T}_1)$, where $\mathcal{T}_1$ is the upper limit topology having all sets $(a, b]$ as basis.

$X_2 = (\mathbb{R}, \mathcal{T}_2)$, where $\mathcal{T}_2 = \{U \subset \mathbb{R} : \mathbb{R} \setminus U \text{ is finite}\} \cup \{\emptyset\}$.

Then

Options
1. X_1 is connected and X_2 is NOT connected
2. both X_1 and X_2 are connected
3. X_1 is NOT connected and X_2 is connected
4. neither X_1 nor X_2 is connected

Question Type : **MCQ**
Question ID : **8232511222**
Status : **Not Answered**
Chosen Option : **--**

Q.50 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) = \begin{cases} \sqrt{x^2+y^2}\ \sin(y^2/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Consider the following statements:

P: f is continuous at $(0,0)$ but f is NOT differentiable at $(0,0)$.

Q: The directional derivative $D_u f(0,0)$ of f at $(0,0)$ exists in the direction of every unit vector $u \in \mathbb{R}^2$.

Then

Options

1. P is TRUE and Q is FALSE
2. both P and Q are TRUE
3. P is FALSE and Q is TRUE
4. both P and Q are FALSE

Question Type : **MCQ**
Question ID : **8232511217**
Status : **Marked For Review**
Chosen Option : **1**

Q.51 The critical point of the differential equation

$$\frac{d^2y}{dt^2} + 2\,\alpha\,\frac{dy}{dt} + \beta^2 y = 0,\ \ \alpha > \beta > 0,$$

is a

Options
1. spiral point and is asymptotically stable
2. saddle point and is unstable
3. node and is asymptotically stable
4. node and is unstable

Question Type : **MCQ**
Question ID : **8232511210**
Status : **Not Answered**
Chosen Option : **--**

Q.52 Consider the following topologies on the set $\mathbb{R}$ of all real numbers:

$\mathcal{T}_1 = \{U \subset \mathbb{R} : 0 \notin U \text{ or } U = \mathbb{R}\}$,

$\mathcal{T}_2 = \{U \subset \mathbb{R} : 0 \in U \text{ or } U = \emptyset\}$,

$\mathcal{T}_3 = \mathcal{T}_1 \cap \mathcal{T}_2$.

Then the closure of the set $\{1\}$ in $(\mathbb{R}, \mathcal{T}_3)$ is

Options
1. $\mathbb{R}$
2. $\{0,1\}$
3. $\mathbb{R}\setminus\{0\}$
4. $\{1\}$

Question Type : **MCQ**
Question ID : **8232511194**
Status : **Not Answered**
Chosen Option : **--**

Q.53

Let $\tilde{x} = \begin{bmatrix} 11/3 \\ 2/3 \\ 0 \end{bmatrix}$ be an optimal solution of the following Linear Programming Problem P:

$$\text{Maximize } 4x_1 + x_2 - 3x_3$$

subject to

$$2x_1 + 4x_2 + ax_3 \leq 10,$$

$$x_1 - x_2 + bx_3 \leq 3,$$

$$2x_1 + 3x_2 + 5x_3 \leq 11,$$

$$x_1 \geq 0,\ x_2 \geq 0 \text{ and } x_3 \geq 0, \text{ where } a,\ b \text{ are real numbers.}$$

If $\tilde{y} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ is an optimal solution of the dual of P, then $p + q + r =$ ________

(round off to two decimal places).

Given Answer : --

Question Type :	**NAT**
Question ID :	**8232511235**
Status :	**Not Answered**

Q.54 Let $C[0,1]$ be the Banach space of real valued continuous functions on $[0,1]$ equipped with the supremum norm. Define $T: C[0,1] \to C[0,1]$ by

$$(Tf)(x) = \int_0^x x\, f(t)\, dt.$$

Let $R(T)$ denote the range space of T. Consider the following statements:

P: T is a bounded linear operator.

Q: $T^{-1}: R(T) \to C[0,1]$ exists and is bounded.

Then

Options
1. P is FALSE and Q is TRUE
2. P is TRUE and Q is FALSE
3. both P and Q are TRUE
4. both P and Q are FALSE

Question Type : **MCQ**
Question ID : **8232511214**
Status : **Not Answered**
Chosen Option : **--**

Q.55 Let $T(z) = \frac{az+b}{cz+d}$, $ad - bc \neq 0$, be the Möbius transformation which maps the points $z_1 = 0$, $z_2 = -i$, $z_3 = \infty$ in the z-plane onto the points $w_1 = 10$, $w_2 = 5 - 5i$, $w_3 = 5 + 5i$ in the w-plane, respectively. Then the image of the set $S = \{z \in \mathbb{C} \,:\, Re(z) < 0\}$ under the map $w = T(z)$ is

Options

1. $\{w \in \mathbb{C} \,:\, |w - 5| < 5\}$
2. $\{w \in \mathbb{C} \,:\, |w| > 5\}$
3. $\{w \in \mathbb{C} \,:\, |w| < 5\}$
4. $\{w \in \mathbb{C} \,:\, |w - 5| > 5\}$

Question Type : **MCQ**
Question ID : **8232511183**
Status : **Answered**
Chosen Option : **2**