COMPLEX ANALYSIS (PYPS)

DECEMBER – 2014

PART – B

1. Let $p(z)=a_0+a_1z+...+a_nz^n$ and $q(z)=b_1z+b_2z^2+...+b_nz^n$ be complex polynomials. If a_0, b_1 are non-zero complex numbers then the residue of p(z)/q(z) at 0 is equal to

1. $\frac{a_0}{b_1}$
2. $\frac{b_1}{a_0}$
3. $\frac{a_1}{b_1}$
4. $\frac{a_0}{a_1}$

2. Let $\sum_{n=1}^{\infty} a_n z^n$ be a convergent power series such that $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=R>0$. Let p be a polynomial of degree d. Then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n p(n) z^n$ equals

1. R
2. d
3. Rd
4. R+d

3. Let f be an entire function on $\mathbb{C}$ and let Ω be a bounded open subset of $\mathbb{C}$. Let $S=\{\text{Re } f(z)+\text{Im } f(z) \mid z\in\Omega\}$. Which of the following statements is/are necessarily correct?

1. S is an open set in $\mathbb{R}$
2. S is a closed set in $\mathbb{R}$
3. S is an open set of $\mathbb{C}$
4. S is a discrete set in $\mathbb{R}$

4. Let $u(x+iy)=x^3-3xy^2+2x$. For which of the following functions v, is u+iv a holomorphic function on $\mathbb{C}$?

1. $v(x+iy)=y^3-3x^2y+2y$
2. $v(x+iy)=3x^2y-y^3+2y$
3. $v(x+iy)=x^3-3xy^2+2x$
4. $v(x+iy)=0$

PART – C

5. Let f be an entire function on $\mathbb{C}$. Let $g(z)=\overline{f(\bar{z})}$. Which of the following statements is/are correct?

1. if $f(z)\in\mathbb{R}$ for all $z\in\mathbb{R}$ then f=g
2. if $f(z)\in\mathbb{R}$ for all $z\in\{z \mid \text{Im } z=0\}\cup\{z \mid \text{Im } z=a\}$, for some a>0, then $f(z+ia)=f(z-ia)$ for all $z\in\mathbb{C}$.
3. If $f(z)\in\mathbb{R}$ for all $z\in\{z \mid \text{Im } z=0\}\cup\{z \mid \text{Im } z=a\}$, for some a>0, then $f(z+2ia)=f(z)$ for all $z\in\mathbb{C}$
4. If $f(z)\in\mathbb{R}$ for all $z\in\{z \mid \text{Im } z=0\}\cup\{z \mid \text{Im } z=a\}$ for some a > 0, then $f(z+ia)=f(z)$ all $z\in\mathbb{C}$

6. Let $f(z)=\sum_{n=0}^{\infty} a_n z^n$ be an entire function and let r be a positive real number. Then

1. $\sum_{n=0}^{\infty}|a_n|^2 r^{2n}\le \sup_{|z|=r}|f(z)|^2$
2. $\sup_{|z|=r}|f(z)|^2\le\sum_{n=0}^{\infty}|a_n|^2 r^{2n}$
3. $\sum_{n=0}^{\infty}|a_n|^2 r^{2n}\le\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^2\,d\theta$
4. $\sup_{|z|=r}|f(z)|^2\le\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^2\,d\theta$

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PART – B

7. $\int_{|z+1|=2}\frac{z^2}{4-z^2}dz=$

1. 0
2. $-2\pi i$
3. $2\pi i$
4. 1

8. How many elements does the set $\left\{z\in C / z^{60}=-1, z^k\neq -1 \text{ for } 0<k<60\right\}$ have?

1. 24
2. 30
3. 32
4. 45

9. Let f be a real valued harmonic function on $\mathbb{C}$, that is, f satisfies the equation $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$.

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Define the functions

$$g = \frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}, \quad h = \frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}$$

Then
1. g and h are both holomorphic functions.
2. g is holomorphic, but h need not be holomorphic.
3. h is holomorphic, but g need not be holomorphic.
4. both g and h are identically equal to the zero function.

PART – C

10. Let f be an entire function. Which of the following statements are correct?
1. f is constant if the range of f is contained in a straight line.
2. f is constant if f has uncountably many zeros.
3. f is constant if f is bounded on {z∈ℂ: Re(z) ≤ 0}.
4. f is constant if the real part of f is bounded.

11. Let f be analytic function defined on the open unit disc in ℂ. Then f is constant if
1. $f\left(\frac{1}{n}\right) = 0 \;\; for\,all\;\; n \geq 1.$
2. $f(z) = 0 \;\; for\,all\;\; |z| = \frac{1}{2}.$
3. $f\left(\frac{1}{n^2}\right) = 0 \;\; for\,all\;\; n \geq 1.$
4. $f(z) = 0 \;\; for\,all\;\; z \in (-1,1).$

12. Let p be a polynomial in 1-complex variable. Suppose all zeroes of p are in the upper half plane $H = \{z \in C \mid \operatorname{Im}(z) > 0\}$. Then
1. $\operatorname{Im}\frac{p'(z)}{p(z)} > 0 \;\; for\; z \in \mathbb{R}.$
2. $\operatorname{Re} i\frac{p'(z)}{p(z)} < 0 \;\; for\; z \in \mathbb{R}$
3. $\operatorname{Im}\frac{p'(z)}{p(z)} > 0 \;\; for\; z \in \mathbb{C},\; with\; \operatorname{Im} z < 0$
4. $\operatorname{Im}\frac{p'(z)}{p(z)} > 0 \;\; for\; z \in \mathbb{C}\, with\; \operatorname{Im} z > 0$

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PART – B

13. Let a, b, c, d ∈ ℝ be such that ad – bc > 0. Consider the Mobius transformation $T_{a,b,c,d}(z) = \frac{az+b}{cz+d}$. Define

$H_+ = \{z \in \mathbb{C} : \operatorname{Im}(z) > 0\}$, $H_- = \{z \in \mathbb{C} : \operatorname{Im}(z) < 0\}$
$R_+ = \{z \in \mathbb{C} : \operatorname{Re}(z) > 0\}$, $R_- = \{z \in \mathbb{C} : \operatorname{Re}(z) < 0\}$.
Then, $T_{a,b,c,d}$ maps
1. H_+ to H_+.
2. H_+ to H_-
3. R_+ to R_+.
4. R_+ to R_-

14. What is the cardinality of the set
$\{z \in \mathbb{C} \mid z^{98} = 1 \text{ and } z^n \neq 1 \text{ for any } 0 < n < 98\}$?
1. 0.
2. 12.
3. 42.
4. 49.

15. Consider the following power series in the complex variable z:
$f(z) = \sum_{n=1}^{\infty} n \log n\, z^n$, $g(z) = \sum_{n=1}^{\infty} \frac{e^{n^2}}{n} z^n$. If r, R are the radii of convergence of f and g respectively, then
1. r = 0, R = 1.
2. r = 1, R = 0.
3. r = 1, R = ∞.
4. r = ∞, R = 1.

PART – C

16. Let $f(z) = \frac{1}{e^z - 1}$ for all z ∈ ℂ such that $e^z \neq 1$. Then
1. f is meromorphic.
2. the only singularities of f are poles.
3. f has infinitely many poles on the imaginary axis.
4. Each pole of f is simple.

17. Let f be an analytic function in ℂ. Then f is constant if the zero set of f contains the sequence
1. $a_n = 1/n$
2. $a_n = (-1)^{n-1}\frac{1}{n}$
3. $a_n = \frac{1}{2n}$
4. $a_n = n$ if 4 does not divide n and $a_n = \frac{1}{n}$ if 4 divides n

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18. Consider the function $f(z)=\frac{1}{z}$ on the annulus $A=\left\{z\in C:\frac{1}{2}<|z|<2\right\}$.
Which of the following is/are true?
1. There is a sequence $\{p_n(z)\}$ of polynomials that approximate f(z) uniformly on compact subsets of A.
2. There is a sequence $\{r_n(z)\}$ of rational functions, whose poles are contained in $\mathbb{C}\backslash A$ and which approximates f(z) uniformly on compact subsets of A.
3. No sequence $\{p_n(z)\}$ of polynomials approximate f(z) uniformly on compact subsets of A.
4. No sequence $\{r_n(z)\}$ of rational functions whose poles are contained in $\mathbb{C}\backslash A$, approximate f(z) uniformly on compact subsets of A.

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PART – B

19. Let P(z), Q(z) be two complex non-constant polynomials of degree m,n respectively. The number of roots of P(z)=P(z)Q(z) counted with multiplicity is equal to:

1. min {m,n} 2. max {m,n}
3. m+n 4. m-n

20. Let D be the open unit disc in C and H(D) be the collection of all holomorphic functions on it.
Let $S=\left\{f\in H(D): f\left(\frac{1}{2}\right)=\frac{1}{2}, f\left(\frac{1}{4}\right)=\frac{1}{4},\ldots, f\left(\frac{1}{2n}\right)=\frac{1}{2n},\ldots\right\}$ and
$T=\left\{f\in H(D): f\left(\frac{1}{2}\right)=f\left(\frac{1}{3}\right)=\frac{1}{2}, f\left(\frac{1}{4}\right)=f\left(\frac{1}{5}\right)=\frac{1}{4},\ldots, f\left(\frac{1}{2n}\right)=f\left(\frac{1}{2n+1}\right)=\frac{1}{2n},\ldots\right\}$ Then
1. Both S,T are singleton sets
2. S is a singleton set but T= ϕ
3. T is a singleton set but S= ϕ
4. Both S,T are empty

21. Let P(x) be a polynomial of degree $d\geq 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty}p(n)z^n$ is

1. 0 2. 1
3. ∞ 4. dependent on d

22. The residue of the function $f(z)=e^{-e^{1/z}}$ at z=0 is:

1. $1+e^{-1}$ 2. e^{-1}
3. $-e^{-1}$ 4. $1-e^{-1}$

PART – C

23. Let H = {z = x + iy $\in\mathbb{C}$: y > 0} be the upper half plane and D = {z $\in\mathbb{C}$: |z| < 1} be the open unit disc. Suppose that f is a Mobius transformation, which maps H conformally onto D. Suppose that f(2i) = 0. Pick each correct statement from below.
1. f has a simple pole at z = -2i.
2. f satisfies $f(i)\overline{f(-i)}=1$.
3. f has an essential singularity at z = - 2i.
4. $|f(2+2i)|=\frac{1}{\sqrt{5}}$.

24. Consider the function $F(z)=\int_1^2\frac{1}{(x-z)^2}dx,$ Im(z) > 0. Then there is a meromorphic function G(z) on $\mathbb{C}$ that agrees with F(z) when Im (z) > 0, such that
1. 1, ∞ are poles of G(z)
2. 0,1,∞ are poles of G(z)
3. 1,2 are poles of G(z)
4. 1,2 are simple poles of G(z).

25. Let f : $\mathbb{C}\to\mathbb{C}$ be an entire function. Suppose that f = u + iv where u,v are the real and imaginary parts of f respectively. Then f is constant if
1. {u(x,y) : z = x + iy$\in\mathbb{C}$} is bounded
2. {v(x,y) : z = x + iy$\in\mathbb{C}$} is bounded
3. {u(x,y) +v(x,y):z = x + iy$\in\mathbb{C}$} is bounded
4. {u^2(x,y) +v^2(x,y):z =x + iy$\in\mathbb{C}$} is bounded

26. Let A = $\{z\in\mathbb{C}\,\|\,z|>1\}$, B = $\{z\in\mathbb{C}\,|\,z\neq 0\}$.
Which of the following are true?
1. There is a continuous onto function f:A → B
2. There is a continuous one to one function f : B → A
3. There is a nonconstant analytic function f : B → A
4. There is a nonconstant analytic function f : A → B

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PART - B

27. The radius of convergence of the series $\sum_{n=1}^{\infty} z^{n^2}$ is

1. 0 2. ∞
3. 1 4. 2

28. Let C be the circle |z| = 3/2 in the complex plane that is oriented in the counter clockwise direction. The value of a for which $\int_C \left(\frac{z+1}{z^2-3z+2}+\frac{a}{z-1}\right)dz = 0$ is

1. 1 2. -1
3. 2 4. -2

29. Suppose f and g are entire functions and g(z) ≠ 0 for all z ∈ ℂ. If |f(z)| ≤ |g(z)|, then we conclude that

1. f(z) ≠ 0 for all z ∈ ℂ.
2. f is a constant function.
3. f(0) = 0.
4. for some C ∈ ℂ, f(z) = Cg(z).

30. Let f be a holomorphic function on 0 < |z| < ϵ, ϵ > 0 given by a convergent Laurent series $\sum_{n=-\infty}^{\infty} a_n z^n$. Given also that $\lim_{z\to 0}|f(z)| = \infty$,
We can conclude that

1. $a_{-1} \neq 0$ and $a_{-n} = 0$ for all n ≥ 2
2. $a_{-N} \neq 0$ for some N ≥ 1 and $a_{-n} = 0$ for all n > N
3. $a_{-n} = 0$ for all n ≥ 1
4. $a_{-n} \neq 0$ for all n ≥ 1

PART - C

31. Let f(z) be the meromorphic function given by $\frac{z}{(1-e^z)\sin z}$. Then

1. z=0 is a pole of order 2.
2. for every k∈ℤ, z=2πik is a simple pole.
3. for every k∈ℤ\{0}, z=kπ is a simple pole.
4. z=π+2πi is a pole.

32. Consider the polynomial
$$P(z)=\sum_{n=1}^{N} a_n z^n, 1\le N<\infty,\ a_n \in \mathbb{R}\backslash\{0\}.$$ Then,
with **D** = {w∈ℂ: |w| < 1}

1. P(D) ⊆ ℝ 2. P(D) is open
3. P(D) is closed 4. P(D) is bounded

33. Consider the polynomial
$$P(z)=\left(\sum_{n=0}^{5} a_n z^n\right)\left(\sum_{n=0}^{9} b_n z^n\right)$$ where $a_n, b_n \in \mathbb{R}$ ∀ n, $a_5 \neq 0$, $b_9 \neq 0$. Then counting roots with multiplicity we can conclude that P(z) has

1. at least two real roots.
2. 14 complex roots
3. no real roots
4. 12 complex roots.

34. Let **D** be the open unit disc in ℂ. Let g: **D** →**D** be holomorphic, g(0)=0, and let
$$h(z) = \begin{cases} \frac{g(z)}{z}, & z \in D, z\neq 0 \\ g'(0), & z = 0 \end{cases}.$$ Which of the following statements are true?

1. h is holomorphic in **D**. 2. h(**D**) ⊆ $\overline{D}$.
3. $|g'(0)|>1$ 4. $\left|g\left(\frac{1}{2}\right)\right| \le \frac{1}{2}$

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PART – B

35. Let C denote the unit circle centered at the origin in ℂ. Then $\frac{1}{2\pi i}\int_C |1+z+z^2|^2\, dz$, where the integral is taken anti-clockwise along C, equals

1. 0 2. 1
3. 2 4. 3

36. Consider the power series
$$f(x)=\sum_{n=2}^{\infty}\log(n)x^n.$$
The radius of convergence of the series $f(x)$ is

1. 0 2. 1 3. 3 4. ∞

37. For an odd integer k ≥ 1, let ℱ be the set of all entire functions f such that $f(x)=|x^k|$ for all $x\in(-1,1)$. Then the cardinality of ℱ is

1. 0
2. 1
3. strictly greater than 1 but finite
4. infinite

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38. Suppose f is holomorphic in an open neighbourhood of $z_0 \in \mathbb{C}$. Given that the series $\sum_{n=0}^{\infty} f^{(n)}(z_0)$ converges absolutely, we can conclude that

1. f is constant
2. f is a polynomial
3. f can be extended to an entire function
4. $f(x) \in \mathbb{R}$ for all $x \in \mathbb{R}$

PART - C

39. Let $f = u + iv$ be an entire function where u,v are the real and imaginary parts of f respectively. If the Jacobian matrix $J_a = \begin{bmatrix} u_x(a) & u_y(a) \\ v_x(a) & v_y(a) \end{bmatrix}$ is symmetric for all $a \in \mathbb{C}$, then

1. f is a polynomial.
2. f is a polynomial of degree ≤1.
3. f is necessarily a constant function
4. f is a polynomial of degree strictly greater than 1.

40. Consider the function $f(z) = \dfrac{\sin(\pi z/2)}{\sin(\pi z)}$.

Then f has poles at
1. all integers
2. all even integers
3. all odd integers
4. all integers of the form 4k+1, $k \in \mathbb{Z}$

41. Consider the Möbius transformation $f(z) = \dfrac{1}{z}, z \in \mathbb{C}$, $z \neq 0$. If C denotes a circle with positive radius passing through the origin, then f map C\{0} to
1. a circle
2. a line
3. a line passing through the origin
4. a line not passing through the origin

42. For which among the following functions f(z) defined on $G = \mathbb{C}\setminus\{0\}$, is there no sequence of polynomials approximating f(z) uniformly on compact subsets of G?

1. exp(z)
2. 1/z
3. z^2
4. $1/z^2$

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PART – B

43. The function $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = e^z + e^{-z}$ has
1. finitely many zeros
2. no zeros
3. only real zeros
4. has infinitely many zeros

44. Let f be a holomorphic function in the open unit disc such that $\lim_{z\to 1} f(z)$ does not exist. Let $\sum_{n=0}^{\infty} a_n z^n$ be the Taylor series of f about z = 0 and let R be its radius of convergence. Then
1. R = 0
2. 0 < R < 1
3. R = 1
4. R > 1

45. Let C be the circle of radius 2 with centre at the origin in the complex plane, oriented in the anti-clockwise direction. Then the integral $\int_C \dfrac{dz}{(z-1)^2}$ is equal to

1. $\dfrac{1}{2\pi i}$
2. $2\pi i$
3. 1
4. 0

46. Let **D** be the open unit disc in the complex plane and $U = \mathbf{D} \setminus \left\{-\frac{1}{2}, \frac{1}{2}\right\}$. Also, let

H_1 = {f : **D**→$\mathbb{C}$ | f is holomorphic and bounded}

and H_2 = {f : **U** → $\mathbb{C}$ | f is holomorphic and bounded}. Then the map r : $H_1 \to H_2$ given by $r(f) = f|_U$, the restriction of f to U, is
1. injective but not surjective
2. surjective but not injective
3. injective and surjective
4. neither injective nor surjective

PART – C

47. Let f be an entire function. Consider $A = \{z \in \mathbb{C} \mid f^{(n)}(z) = 0$ for some positive integer n}. Then

1. if A = $\mathbb{C}$, then f is a polynomial
2. if A = $\mathbb{C}$, then f is a constant function
3. if A is uncountable, then f is a polynomial
4. If A is uncountable, then f is a constant function

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48. Let f:ℂ→ℂ be a holomorphic function and let u be the real part of f and v the imaginary part of f. Then, for x,y∈ℝ. $|f'(x+iy)|^2$ it equal to

1. $u_x^2 + u_y^2$
2. $u_x^2 + v_x^2$
3. $v_y^2 + u_y^2$
4. $v_y^2 + v_x^2$

49. Let $p(z) = z^n + a_{n-1} z^{n-1} + ... + a_0$, where $a_0, ..., a_{n-1}$ are complex numbers and let $q(z) = 1 + a_{n-1} z + ... + a_0 z^n$. If $|p(z)| \le 1$ for all z with $|z| \le 1$ then

1. $|q(z)| \le 1$ for all z with $|z| \le 1$
2. q(z) is a constant polynomial
3. $p(z) = z^n$ for all complex numbers z
4. p(z) is a constant polynomial

50. Let f be a non-constant entire function and let E be the image of f. Then

1. E is an open set
2. E ∩ {z:|z|<1} is empty
3. E ∩ℝ is non – empty
4. E is a bounded set

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PART – B

51. The value of the integral $\oint_{|1-z|=1} \frac{e^z}{z^2-1} dz$ is

1. 0
2. $(\pi i)e$
3. $(\pi i)e - (\pi i)e^{-1}$
4. $(e + e^{-1})$

52. Let $f : \{z / |z| < 1\} \to \mathbb{C}$ be a non-constant analytic function. Which of the following conditions can possibly be satisfied by f ?

1. $f\left(\frac{1}{n}\right) = f\left(\frac{-1}{n}\right) = \frac{1}{n^2} \ \forall \ n \in \mathbb{N}$
2. $f\left(\frac{1}{n}\right) = f\left(\frac{-1}{n}\right) = \frac{1}{2n+1} \ \forall \ n \in \mathbb{N}$
3. $\left| f\left(\frac{1}{n}\right) \right| < 2^{-n} \ \forall \ n \in \mathbb{N}$
4. $\frac{1}{\sqrt{n}} < \left| f\left(\frac{1}{n}\right) \right| < \frac{2}{\sqrt{n}} \forall \ n \in \mathbb{N}$

53. Consider the map $\varphi : \mathbb{C} \backslash \{1\} \to \mathbb{C}$ given by $\varphi(z) = \frac{1+z}{1-z}$. Which of the following is true?

1. $\varphi(\{z \in \mathbb{C} \mid |z| < 1\}) \subseteq \{z \in \mathbb{C} \mid |z| < 1\}$
2. $\varphi(\{z \in \mathbb{C} \mid Re(z) < 0\}) \subseteq \{z \in \mathbb{C} \mid Re(z) < 0\}$
3. φ is onto
4. $\varphi(\mathbb{C} \backslash \{1\}) = \mathbb{C} \backslash \{-1\}$

54. Suppose that f is a non constant analytic function defined over ℂ. Then which of the following is false?

1. f is unbounded
2. f sends open sets into open sets
3. there exists an open connected domain U on which f is never f|u attains its minimum at one point of u
4. the image of f is dense in ℂ

PART – C

55. Let Ω be an open connected subset of ℂ. Let $E = \{z_1, z_2, \dots, z_r\} \subseteq \Omega$. Suppose that f : Ω → ℂ is a function such that $f_{|(\Omega \backslash E)}$ is analytic. Then f is analytic on Ω if

1. f is continuous on Ω
2. f is bounded on Ω
3. for every j, if $\sum_{m \in Z} a_m (z - z_j)^m$ is Laurent series expansion of f at z_j, then $a_m = 0$ for m = -1, -2, -3, …
4. for every j, if $\sum_{m \in Z} a_m (z - z_j)^m$ is Laurent series expansion of f at z_j, then $a_{-1} = 0$

56. Suppose that f: ℂ → ℂ is an analytic function. Then f is a polynomial if

1. for any point a ∈ ℂ, if $f(z) = \sum_0^\infty a_n (z-a)^n$ is a power series expansion at a, then $a_n = 0$ for at least one n
2. $\lim_{|z| \to \infty} |f(z)| = \infty$
3. $\lim_{|z| \to \infty} |f(z)| = M$ for some M
4. $|f(z)| \le M|z|^n$ for |z| sufficiently large and for some n

57. Let **D** be the open unit disk centered at 0 in ℂ and f: **D** → ℂ be an analytic function. Let f = u + iv, where u, v are the real and imaginary parts of f. If $f(z) = \sum a_n z^n$ is the power series of f, then f is constant if

1. $\bar{f}$ is analytic
2. $u(1/2) \ge u(z) \ \forall \ z \in$ **D**
3. The set $\{n \in \mathbb{N} | a_n = 0\}$ is infinite

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4. For any closed curve γ in **D**, $\int_\gamma \frac{f(z)\,dz}{(z-a)^2} = 0\ \forall$ a $\in$ **D** with |a| $\geq \frac{1}{2}$

58. Which of the following statements are true?

1. If $\{a_k\}$ is bounded then $\sum_0^\infty a_k z^k$ defines an analytic function on the open unit disk
2. If $\sum_0^\infty a_k z^k$ defines an analytic function on the open disk then $\{a_k\}$ must converge to zero
3. If $f(z)=\sum_0^\infty a_k z^k$ and $g(z)=\sum_0^\infty b_k z^k$ are two power series functions whose radii of convergence are 1, then the product f.g has a power series representation of the form $\sum_0^\infty c_k z^k$ on the open unit disk
4. If $f(z)=\sum_0^\infty a_k z^k$ has a radius of convergence 1, then f is continuous on $\Omega = \{z \in \mathbb{C} \mid |z| \leq 1\}$

DECEMBER – 2018

PART – B

59. Consider the polynomials p(z), q(z) in the complex variable z and let $I_{p,q} = \oint_\gamma p(z)\overline{q(z)}dz$, where γ denotes the closed contour $\gamma(t)=e^{it}, 0\leq t \leq 2\pi$. Then

1. $I_{z^m,z^n} = 0$ for all positive integers m,n with m≠n
2. $I_{z^n,z^n} = 2\pi i$ for all positive integers n
3. $I_{p,1} = 0$ for all polynomials p
4. $I_{p,q} = p(0)\overline{q(0)}$ for all polynomials p,q

60. Let $\gamma(t)=3e^{it}, 0\leq t\leq 2\pi$ be the positively oriented circle of radius 3 centred at the origin. The value of λ for which $\oint_\gamma \frac{\lambda}{z-2}dz = \oint_\gamma \frac{1}{z^2-5z+4}dz$ is

1. $\lambda = \frac{-1}{3}$
2. $\lambda = 0$
3. $\lambda = \frac{1}{3}$
4. $\lambda = 1$

61. Let f: $\mathbb{C} \to \mathbb{C}$ be a non-constant entire function and let Image (f) = {w$\in\mathbb{C}$:$\exists$ z$\in\mathbb{C}$ such that f(z)=w}. Then

1. The interior of Image (f) is empty
2. Image (f) intersects every line passing through the origin
3. There exists a disc in the complex plane, which is disjoint from Image (f)
4. Image (f) contains all its limit points

PART - C

62. Let H denote the upper half plane, that is, H = {z = x + iy : y > 0}. For z $\in$ H, which of the following are true?

1. $\frac{1}{z} \in H$
2. $\frac{1}{z^2} \in H$
3. $\frac{-z}{z+1} \in H$
4. $\frac{z}{2z+1} \in H$

63. Let f : $\mathbb{C} \to \mathbb{C}$ be an analytic function. Then which of the following statements are true?

1. If |f(z)| $\leq$ 1 for all z $\in$ $\mathbb{C}$, then f ′ has infinitely many zeroes in $\mathbb{C}$
2. If f is onto, then the function f (cos z) is onto
3. If f is onto, then the function f(e^z) is onto
4. If f is one-one, then the function f(z^4 + z + 2) is one-one

64. Consider the entire functions f(z) = 1+z +z^{20} and g(z) = e^z, z$\in\mathbb{C}$. Which of the following statements are true ?

1. $\lim_{|z|\to\infty} |f(z)| = \infty$
2. $\lim_{|z|\to\infty} |g(z)| = \infty$
3. f^{-1}({z$\in\mathbb{C}$: |z|≤R}) is bounded for every R>0
4. g^{-1}({z$\in\mathbb{C}$: |z|≤R}) is bounded for every R>0

65. Which of the following statements are true?

1. tan z is an entire function
2. tan z is a meromorphic function on $\mathbb{C}$
3. tan z has an isolated singularity at ∞
4. tan z has a non-isolated singularity at ∞

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PART – B

66. Let C be the counter-clockwise oriented circle of radius $\frac{1}{2}$ centred at $i = \sqrt{-1}$. Then the value of the contour integral $\oint_C \frac{dx}{x^4 - 1}$ is

1. $-\pi/2$
2. $\pi/2$
3. $-\pi$
4. π

67. Consider the function $f : \mathbb{C} \to \mathbb{C}$ given by $f(z) = e^z$. Which of the following is false?

1. $f(\{z \in \mathbb{C} : |z| < 1\})$ is not an open set
2. $f(\{z \in \mathbb{C} : |z| \le 1\})$ is not an open set
3. $f(\{z \in \mathbb{C} : |z| = 1\})$ is a closed set
4. $f(\{z \in \mathbb{C} : |z| > 1\})$ is an unbounded open set

68. Given a real number $a > 0$, consider the triangle Δ with vertices 0, a, a + ia. If Δ is given the counter clockwise orientation, then the contour integral $\oint_\Delta \mathrm{Re}(z)\,dz$ (with Re (z) denoting the real part of z) is equal to

1. 0
2. $i\frac{a^2}{2}$
3. ia^2
4. $i\frac{3a^2}{2}$

69. Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function such that $\lim_{x\to 0}\left|f\left(\frac{1}{x}\right)\right| = \infty$. Then which of the following is true?

1. f is constant
2. f can have infinitely many zeros
3. f can have at most finitely many zeros
4. f is necessarily nowhere vanishing

PART – C

70. Let $f(z) = (z^3 + 1)\sin z^2$ for $z \in \mathbb{C}$. Let $f(z) = u(x, y) + i\,v(x, y)$, where $z = x + iy$ and u, v are real valued functions. Then which of the following are true?

1. $u : \mathbb{R}^2 \to \mathbb{R}$ is infinitely differentiable
2. u is continuous but need not be differentiable
3. u is bounded
4. f can be represented by an absolutely convergent power series $\sum_{n=0}^{\infty} a_n z^n$ for all $z \in \mathbb{C}$

71. Let Re(z), Im(z) denote the real and imaginary parts of $z \in \mathbb{C}$, respectively. Consider the domain $\Omega = \{z \in \mathbb{C} : \mathrm{Re}(z) > |\mathrm{Im}(z)|\}$ and let $f_n(z) = \log z^n$, where $n \in \{1, 2, 3, 4\}$ and where $\log : \mathbb{C} \setminus (-\infty, 0] \to \mathbb{C}$ defines the principal branch of logarithm. Then which of the following are true?

1. $f_1(\Omega) = \{z \in \mathbb{C} : 0 \le |\mathrm{Im}(z)| < \pi/4\}$
2. $f_2(\Omega) = \{z \in \mathbb{C} : 0 \le |\mathrm{Im}(z)| < \pi/2\}$
3. $f_3(\Omega) = \{z \in \mathbb{C} : 0 \le |\mathrm{Im}(z)| < 3\pi/4\}$
4. $f_4(\Omega) = \{z \in \mathbb{C} : 0 \le |\mathrm{Im}(z)| < \pi\}$

72. Consider the set

$F = \{f : \mathbb{C} \to \mathbb{C} \mid f$ is an entire function, $|f'(z)| \le |f(z)|$ for all $z \in \mathbb{C}\}$.

Then which of the following are true?

1. F is a finite set
2. F is an infinite set
3. $F = \{\beta e^{\alpha z} : \beta \in \mathbb{C}, \alpha \in \mathbb{C}\}$
4. $F = \{\beta e^{\alpha z} : \beta \in \mathbb{C}, |\alpha| \le 1\}$

73. Let $D = \{z \in \mathbb{C} \mid |z| < 1\}$ and $\omega \in D$. Define $F_\omega : D \to D$ by $F_\omega(z) = \frac{\omega - z}{1 - \overline{\omega} z}$. Then which of the following are true?

1. F is one to one
2. F is not one to one
3. F is onto
4. F is not onto

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PART – B

74. For $z \in \mathbb{C}$, let $f(z) = \begin{cases} \frac{\overline{z}^2}{z} & \text{if } z \ne 0. \\ 0 & \text{otherwise.} \end{cases}$

Then which of the following statements is false?

1. f(z) is continuous everywhere
2. f(z) is not analytic in any open neighbourhood of zero
3. zf(z) satisfies the Cauchy-Riemann equations at zero
4. f(z) is analytic in some open subset of $\mathbb{C}$

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75. Let T : $\mathbb{C} \to M_2(\mathbb{R})$ be the map given by

$$T(z) = T(x+iy) = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$$

Then which of the following statements is false?

1. $T(z_1z_2) = T(z_1)\,T(z_2)$ for all $z_1, z_2 \in \mathbb{C}$
2. T(z) is singular if and only if z = 0
3. There does not exist non-zero A $\in M_2(\mathbb{R})$ such that the trace of T(z)A is zero for all $z \in \mathbb{C}$
4. $T(z_1 + z_2) = T(z_1) + T(z_2)$ for all $z_1, z_2 \in \mathbb{C}$

76. Consider the polynomial $f(z) = z^2 + az + p^{11}$, where $a \in Z\backslash\{0\}$ and $p \geq 13$ is a prime. Suppose that $a^2 \leq 4p^{11}$. Which of the following statements is true?

1. f has a zero on the imaginary axis
2. f has a zero for which the real and imaginary parts are equal
3. f has distinct roots
4. f has exactly one real root

77. Let f : $\mathbb{C} \to \mathbb{C}$ be an entire function with $f\left(\frac{1}{n}\right) = \frac{1}{n^2}$ for all n $\in \mathbb{N}$. Then which of the following statements is true?

1. No such f exists
2. such an f is not unique
3. $f(z) = z^2$ for all $z \in \mathbb{C}$
4. f need not be a polynomial function

PART – C

78. Let U be an open subset of $\mathbb{C}$ and f : U $\to \mathbb{C}$ be an analytic function. Then which of the following are true?

1. If f is one-one, then f(U) is open in $\mathbb{C}$
2. If f is onto, then U = $\mathbb{C}$
3. If f is onto, then f is one-one
4. f(U) is closed in $\mathbb{C}$, then f(U) is connected

79. Let f : $\mathbb{C} \to \mathbb{C}$ be an analytic function. For $z_0 \in \mathbb{C}$, which of the following statements are true?

1. can take the value z_0 at finitely many points in $\frac{1}{n} \mid n \in \mathbb{N}$
2. $f(1/n) = z_0$ for all n $\in \mathbb{N} \Rightarrow$ f is the constant function z_0
3. $f(n) = z_0$ for all n $\in \mathbb{N} \Rightarrow$ f is the constant function z_0
4. $f(r) = z_0$ for all r $\in \mathbb{Q} \cap [1, 2] \Rightarrow$ f is the constant function z_0

80. Let U $\subset \mathbb{C}$ be an open connected set and f: U $\to \mathbb{C}$ be a non-constant analytic function. Consider the following two sets:

X = {z $\in$ U : f(z) = 0}

Y = {z $\in$ U : f vanishes on an open neighbourhood of z in U}

Then which of the following statements are true?

1. X is closed in U
2. Y is closed in U
3. X has empty interior
4. Y is open in U

81. Consider the power series $f(z) = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n)!}$. Which of the following are true?

1. Radius of convergence of f(z) is infinite
2. The set {f(x) : x $\in \mathbb{R}$} is bounded
3. The set {f(x) : -1 < x < 1} is bounded
4. f(z) has infinitely many zeroes

JUNE – 2020

PART – B

82. Let γ be the positively oriented circle in the complex plane given by {z $\in \mathbb{C}$: |z – 1| = 1}.

Then $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z^3 - 1}$ equals

1. 3 2. 1/3
3. 2 4. 1/2

83. For a positive integer p, consider the holomorphic function

$f(z)=\frac{\sin z}{z^p}$ for $z \in \mathbb{C}\backslash\{0\}$.

For which values of p does there exist a holomorphic function $g : \mathbb{C}\backslash\{0\} \to \mathbb{C}$ such that $f(z) = g'(z)$ for $z \in \mathbb{C}\backslash\{0\}$?
1. All even integers
2. All odd integers
3. All multiples of 3
4. All multiples of 4

84. Let γ be the positively oriented circle in the complex plane given by $\{z \in \mathbb{C} : |z - 1| = 1/2\}$. The line integral $\int_\gamma \frac{ze^{1/z}}{z^2-1}dz$ equals

1. $i\pi e$ 2. $-i\pi e$
3. πe 4. $-\pi e$

85. Let p be a positive integer. Consider the closed curve $r(t) = e^{it}$, $0 \le t < 2\pi$. Let f be a function holomorphic in $\{z: |z| < R\}$ where $R > 1$. If f has a zero only at z_0, $0 < |z_0| < R$, and it is of multiplicity q, then $\frac{1}{2\pi i}\int_r \frac{f'(z)}{f(z)} z^p dz$ equals

1. qz_0^p 2. $z_0 q^p$
3. pz_0^q 4. $z_0 p^q$

PART – C

86. For $z \neq -i$, let $f(z) = \exp\left(\frac{1}{z+i}\right) - 1$. Which of the following are true?
1. f has finitely many zeroes
2. f has a sequence of zeroes that converges to a removable singularity of f
3. f has a sequence of zeroes that converges to a pole of f
4. f has a sequence of zeroes that converges to an essential singularity of f

87. Let f be a holomorphic function on the open unit disc

$\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. Suppose that $|f| \ge 1$ on $\mathbb{D}$ and $f(0) = i$.

Which of the following are possible values of $f\left(\frac{1}{2}\right)$?

1. $-i$ 2. i
3. 1 4. -1

88. Let $\mathbb{D} = \{z \in \mathbb{C}: |z| < 1\}$ be the open unit disc and let $f : \mathbb{D} \to \mathbb{D}$ be a holomorphic function. Suppose that $f(0) = 0$ and $f'(0) = 0$. Which of the following are possible values of $f\left(\frac{1}{2}\right)$?

1. 1/4 2. -1/4
3. 1/3 4. -1/3

89. Let n be a positive integer. For a real number $R > 1$, let $z(\theta) = Re^{i\theta}$, $0 \le \theta < 2\pi$.
The set
$\{\theta \in [0, 2\pi) : |z(\theta)^n + 1| = |z(\theta)|^n - 1\}$
contains which of the following sets?
1. $\{\theta \in [0, 2\pi): \cos n\theta = 1\}$
2. $\{\theta \in [0, 2\pi): \sin n\theta = 1\}$
3. $\{\theta \in [0, 2\pi): \cos n\theta = -1\}$
4. $\{\theta \in [0, 2\pi): \sin n\theta = -1\}$

JUNE – 2021

PART – B

90. Let f(z) be a non-constant entire function and $z = x + iy$. Let u(x, y), v(x, y) denote its real and imaginary parts respectively. Which of the following statements is FALSE?
1. $u_x = v_y$ and $u_y = -v_x$
2. $u_y = v_x$ and $u_x = -v_y$
3. $|f'(x + iy)|^2 = u_x(x, y)^2 + v_x(x, y)^2$
4. $|f'(x + iy)|^2 = u_y(x, y)^2 + v_y(x, y)^2$

91. Let f be a rational function of a complex variable z given by $f(z)=\frac{z^3+2z-4}{z}$.
The radius of convergence of the Taylor series of f at z = 1 is
1. 0 2. 1
3. 2 4. ∞

92. Let γ be the positively oriented circle $\{z \in \mathbb{C}: |z| = 3/2\}$.
Suppose that

$$\int_\gamma \frac{e^{i\pi z}}{(z-1)(z-2i)^2}dz = 2\pi i C.$$

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Then |C| equals
1. 2 2. 5
3. 1/2 4. 1/5

93. Let $\mathbb{D}\subset\mathbb{C}$ be the open disc $\{z \in\mathbb{C}: |z| < 1\}$ and $O(\mathbb{D})$ be the space of all holomorphic functions on $\mathbb{D}$. Consider the sets

$$A = \left\{f \in O(\mathbb{D}): f\left(\frac{1}{n}\right) = \begin{cases} e^{-n} & if\ n\, is\, even \\ 0 & if\ n\, is\, odd \end{cases};\ for\ n\geq 2\right\}$$

$B = \{f \in O(\mathbb{D}): f(1/n) = (n - 2)/(n - 1), n \geq 2\}$.
Which of the following statements is true?
1. Both A and B are non-empty
2. A is empty and B has exactly one element
3. A has exactly one element and B is empty
4. Both A, B are empty

PART – C

94. For any complex valued function f, let D_f denote the set on which the function f satisfies Cauchy-Riemann equations. Identify the functions for which D_f is equal to $\mathbb{C}$.

(1) $f(z) = \dfrac{z}{1+|z|}$

(2) $f(z) = (\cos \alpha x - \sin \alpha y) + i(\sin \alpha x + \cos \alpha y)$, where $z = x + iy$

(3) $f(z) = \begin{cases} e^{-\frac{1}{z^4}} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$

(4) $f(z) = x^2 + iy^2$, where $z = x + iy$

95. Let $\mathbb{T}$ denote the unit circle $\{z \in\mathbb{C}: |z| = 1\}$ in the complex plane and let $\mathbb{D}$ be the open unit disc $\{z \in\mathbb{C}: |z| < 1\}$. Let R denote the set of points z_0 in $\mathbb{T}$ for which there exists a holomorphic function f in an open neighbourhood U_{z_0} of z_0 such that $f(z) = \sum_{n=0}^{\infty} z^{4n}$ in $U_{z_0} \cap \mathbb{D}$. Then R contains
(1) All points of $\mathbb{T}$
(2) Infinitely many points of $\mathbb{T}$
(3) All points of $\mathbb{T}$ except a finite set
(4) No points of $\mathbb{T}$

96. Consider the function

$$f(z) = \frac{(\sin z)^m}{(1-\cos z)^n}\ for\ 0<|z|<1 \text{ where}$$

m, n are positive integers. Then z = 0 is
(1) A removable singularity if $m \geq 2n$
(2) A pole if $m < 2n$
(3) A pole if $m \geq 2n$
(4) An essential singularity for some values of m, n

97. Let f be an entire function such that $|zf(z) - 1 + e^z| \leq 1 + |z|$ for all $z \in\mathbb{C}$. Then
(1) $f'(0) = -1$
(2) $f'(0) = -1/2$
(3) $f''(0) = -1/3$
(4) $f''(0) = -1/4$

JUNE – 2022

PART – B

98. If $|e^{e^z}| = 1$ for a complex number $z=x + iy$, $x, y \in \mathbb{R}$, then which of the following is true?
1. $x = n\pi$ for some integer n
2. $y = (2n+1)\dfrac{\pi}{2}$ for some integer n
3. $y = n\pi$ for some integer n
4. $x = (2n+1)\dfrac{\pi}{2}$ for some integer n

99. Let $f(z) = (1-z)e^{\left(z+\frac{z^2}{2}\right)} = 1 + \sum_{n=1}^{\infty} a_n z^n$. Which of the following is false?
1. $f'(z) = -z^2 e^{\left(z+\frac{z^2}{2}\right)}$
2. $a_1 = a_2$
3. $a_n \in (-\infty, 0]$
4. $\sum_{n=3}^{\infty} |a_n| < 1$

100. Let f be a non-constant entire function such that $|f(z)| = 1$ for $|z| = 1$. Let U denote the open unit disk around 0. Which of the following is False?
1. $f(\mathbb{C}) = \mathbb{C}$
2. f has atleast one zero in U
3. f has atmost finitely many distinct zeroes in $\mathbb{C}$
4. f can have a zero outside U

101. For a positive integer n, let $f^{(n)}$ denote the n^{th} derivative of f. Suppose an entire function f satisfies $f^{(2)} + f = 0$. Which of the following is correct?
1. $(f^{(n)}(0))_{n\geq 1}$ is convergent
2. $\lim_{n\to\infty} f^{(n)}(0) = 1$
3. $\lim_{n\to\infty} f^{(n)}(0) = -1$
4. $(|f^{(n)}(0)|)_{n\geq 1}$ has a convergent subsequence

PART – C

102. For a bounded open connected subset Ω of $\mathbb{C}$, let $f : \Omega \to \mathbb{C}$ be holomorphic. Let (z_k) be a sequence of distinct complex numbers in Ω converging to z_0. If $f(z_k) = 0$ for all $k \geq 1$ then which of the following statements are necessarily true?
1. If f is non-zero, then $z_0 \in \partial\Omega$
2. There exists r > 0 such that f(z) = 0 for every $z \in \Omega$ satisfying $|z - z_0| \leq r$
3. If $z_0 \in \Omega$, there exists r > 0 such that f(z) = 0 on $|z - z_0| = r$
4. $z_0 \in \partial\Omega$

103. Let f be an entire function such that $f(z)^2 + f'(z)^2 = 1$. Consider the following sets
$X = \{z : f'(z) = 0\}$, $Y = \{z : f''(z) + f(z) = 0\}$.
Which of the following statements are true?
1. Either X or Y has a limit point
2. If Y has a limit point, then f′ is constant
3. If X has a limit point, then f is constant
4. $f(z) \in \{1, -1\}$ for all $z \in \mathbb{C}$

104. Let U be a bounded open set of $\mathbb{C}$ containing 0. Let $f : U \to U$ be holomorphic with f(0) = 0. For $n \in \mathbb{N}$, let f^n denote the composition of f done n times, that is, $f^n = \underbrace{f \circ \ldots \circ f}_{n\ times}$ while f′ denotes the derivative of f. Which of the following statements are true?
1. $(f^n)'(0) = (f'(0))^n$
2. $f^n(U) \subset U$
3. The sequence $((f'(0))^n)_n$ is bounded
4. $|f'(0)| \leq 1$

105. For an open subset Ω of $\mathbb{C}$ such that $0 \in \Omega$, which of the following statements are true?
1. $\{e^z : z \in \Omega\}$ is an open subset of $\mathbb{C}$
2. $\{|e^z| : z \in \Omega\}$ is an open subset of $\mathbb{R}$
3. $\{\sin z : z \in \Omega\}$ is an open subset of $\mathbb{C}$
4. $\{|\sin z| : z \in \Omega\}$ is an open subset of $\mathbb{R}$

JUNE – 2023

PART – B

106. Let C be the positively oriented circle in the complex plane of radius 3 centered at the origin. What is the value of the integral $\int_C \frac{dz}{z^2(e^z - e^{-z})}$?

(1) $i\pi/12$ (2) $-i\pi/12$
(3) $i\pi/6$ (4) $-i\pi/6$

107. Consider the function f defined by $f(z) = \frac{1}{1 - z - z^2}$ for $z \in \mathbb{C}$ such that $1 - z - z^2 \neq 0$. Which of the following statements is true?
(1) f is an entire function
(2) f has a simple pole at z = 0
(3) f has a Taylor series expansion $f(z) = \sum_{n=0}^{\infty} a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0 = 1$, $a_1 = 0$ and $a_{n+2} = a_n + a_{n+1}$ for $n \geq 0$
(4) f has a Taylor series expansion $f(z) = \sum_{n=0}^{\infty} a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0 = 1$, $a_1 = 1$ and $a_{n+2} = a_n + a_{n+1}$ for $n \geq 0$

108. Let f be an entire function that satisfies $|f(z)| \leq e^y$ for all $z = x + iy \in \mathbb{C}$, where $x, y \in \mathbb{R}$. Which of the following statements is true?
(1) $f(z) = ce^{-iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(2) $f(z) = ce^{iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(3) $f(z) = e^{-ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(4) $f(z) = e^{ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$

109. Let $f(z) = \exp\left(z + \frac{1}{z}\right)$, $z \in \mathbb{C}\setminus\{0\}$. The residue of f at z = 0 is
(1) $\sum_{l=0}^{\infty} \frac{1}{(l+1)!}$
(2) $\sum_{l=0}^{\infty} \frac{1}{l!(l+1)}$

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(3) $\sum_{l=0}^{\infty} \frac{1}{l!(l+1)!}$

(4) $\sum_{l=0}^{\infty} \frac{1}{(l^2+l)!}$

PART – C

110. Let f(z) be an entire function on ℂ. Which of the following statements are true?
(1) $f(\bar{z})$ is an entire function
(2) $\overline{f(z)}$ is an entire function
(3) $\overline{f(\bar{z})}$ is an entire function
(4) $\overline{f(z)} + f(\bar{z})$ is an entire function

111. Let 𝔻 = {z ∈ ℂ: |z| < 1} be the open unit disc and C the positively oriented boundary {|z| = 1}. Fix a finite set $\{z_1, z_2, \ldots, z_n\} \subseteq$ 𝔻 of distinct points and consider the polynomial
$g(z) = (z - z_1)(z - z_2) \ldots (z - z_n)$
of degree n. Let f be a holomorphic function in an open neighbourhood of $\bar{𝔻}$ and define

$$P(z) = \frac{1}{2\pi i}\int_C f(\zeta)\frac{g(\zeta)-g(z)}{(\zeta - z)g(\zeta)}d\zeta.$$

Which of the following statements are true?
(1) P is a polynomial of degree n
(2) P is a polynomial of degree n – 1
(3) P is a rational function on ℂ with poles at $z_1, z_2, \ldots, z_n$
(4) $P(z_j) = f(z_j)$ for j = 1, 2, …, n.

112. Let D = {z ∈ ℂ: |z| < 1}. Consider the following statements.
(a) f : D → D be a holomorphic function. Suppose α, β are distinct complex numbers in D such that f(α) = α and f(β) = β. Then f(z) = z for all z ∈ D.
(b) There does not exist a bijective holomorphic function from D to the set of all complex numbers whose imaginary part is positive.
(c) f : D → D be a holomorphic function. Suppose α ∈ D be such that f(α) = α and f′(α) = 1. Then f(z) = z for all z ∈ D.
Which of the following options are true?
(1) (a), (b) and (c) are all true.
(2) (a) is true.
(3) Both (a) and (b) are false.
(4) Both (a) and (c) are true.

113. Let f : {z : |z| < 1} → {z : |z| ≤ 1/2} be a holomorphic function such that f(0) = 0. Which of the following statements are true?
(1) |f(z)| ≤ |z| for all z in {z : |z| < 1}.
(2) $|f(z)| \le \left|\frac{z}{2}\right|$ for all z in {z : |z| < 1}
(3) |f(z)| ≤ 1/2 for all z in {z : |z| < 1}
(4) It is possible that f(1/2) = 1/2.

DECEMBER – 2023

PART – B

114. Let ℍ = {z ∈ ℂ: Im(z) > 0} denote the upper half plane and let f : ℂ → ℂ be defined by $f(z) = e^{iz}$. Which one of the following statements is true?
(1) f(ℍ) = ℂ\{0}.
(2) f(ℍ) ∩ ℍ is countable.
(3) f(ℍ) is bounded.
(4) f(ℍ) is a convex subset of ℂ.

115. How many roots does the polynomial
$z^{100} - 50z^{30} + 40z^{10} + 6z + 1$
have in the open disc {z ∈ ℂ : |z| < 1}?
(1) 100 (2) 50
(3) 30 (4) 0

116. Let f be a meromorphic function on an open set containing the unit circle C and its interior. Suppose that's f has no zeros and no poles on C and let n_p and n_0 denote the number of poles and zeros of f inside C respectively. Which one of the following is true?
(1) $\frac{1}{2\pi i}\int_C \frac{(zf)'}{zf}dz = n_0 - n_p + 1.$
(2) $\frac{1}{2\pi i}\int_C \frac{(zf)'}{zf}dz = n_0 - n_p - 1.$
(3) $\frac{1}{2\pi i}\int_C \frac{(zf)'}{zf}dz = n_0 - n_p.$
(4) $\frac{1}{2\pi i}\int_C \frac{(zf)'}{zf}dz = n_p - n_0.$

117. Let f : ℂ → ℂ be a real-differentiable function. Define u, v : $\mathbb{R}^2 \to \mathbb{R}$ by u(x, y) =

Re f(x + iy) and v(x, y) = Im f(x + iy), x, y ∈ ℝ.

Let ∇u = (u_x, u_y) denote the gradient. Which one of the following is necessarily true?

(1) For c_1, c_2 ∈ ℂ, the level curves u = c_1 and v = c_2 are orthogonal wherever they intersect.

(2) ∇u . ∇v = 0 at every point.

(3) If f is an entire function, then ∇u . ∇v = 0 at every point.

(4) If ∇u . ∇v = 0 at ever point, then f is an entire function.

PART – C

118. Let $\Omega_1 = \{z \in \mathbb{C} : |z| < 1\}$ and $\Omega_2 = \mathbb{C}$. Which of the following statements are true?

(1) There exists a holomorphic surjective map $f : \Omega_1 \to \Omega_2$.

(2) There exists a holomorphic surjective map $f : \Omega_2 \to \Omega_1$.

(3) There exists a holomorphic injective map $f : \Omega_1 \to \Omega_2$.

(4) There exists a holomorphic injective map $f : \Omega_2 \to \Omega_1$.

119. For every n ≥ 1, consider the entire function $p_n(z) = \sum_{k=0}^{n} \frac{z^k}{k!}$. Which of the following statements are true?

(1) The sequence of functions $(p_n)_{n\geq 1}$ converges to an entire function uniformly on compact subsets of ℂ.

(2) For all n ≥ 1, p_n has a zero in the set $\{z \in \mathbb{C} : |z| \leq 2023\}$.

(3) There exists a sequence (z_n) of complex numbers such that $\lim_{n\to\infty} |z_n| = \infty$ and $p_n(z_n) = 0$ for all n ≥ 1.

(4) Let S_n denote the set of all the zeros of p_n. If $a_n = \min_{z \in S_n} |z|$, then $a_n \to \infty$ as $n \to \infty$.

120. Let X be an uncountable subset of ℂ and let f : ℂ → ℂ be an entire function. Assume that for every z ∈ X, there exists an integer n ≥ 1 such that $f^{(n)}(z) = 0$. Which of the following statements are necessarily true?

(1) f = 0.

(2) f is a constant function.

(3) There exists a compact subset K of ℂ such that $f^{-1}(K)$ is not compact.

(4) f is a polynomial.

121. For an integer k, consider the contour integral $I_k = \int_{|z|=1} \frac{e^z}{z^k} dz.$ Which of the following statements are true?

(1) $I_k = 0$ for every integer k.

(2) $I_k \neq 0$ if k ≥ 1.

(3) $|I_k| \leq |I_{k+1}|$ for every integer k.

(4) $\lim_{k\to\infty} |I_k| = \infty$.

Phone: 9872311001 e–mail: mathsmim@gmail.com web: www.onlinemim.com

ANSWERS

1. (1)	2. (1)	3.
4. (2)	5. (1,2,3)	6. (1,3)
7. (3)	8. (3)	9. (2)
10. (1,2,4)	11. (1,2,3,4)	12. (1,2,3)
13. (1)	14. (3)	15. (2)
16. (1,2,3,4)	17. (1,2,3,4)	18. (2,3)
19. (3)	20. (2)	21. (2)
22. (3)	23. (1,2,4)	24. (3,4)
25. (1,2,3,4)	26. (1,2,4)	27. (3)
28. (3)	29. (4)	30. (2)
31. (2,3)	32. (2,4)	33. (1,2)
34. (1,2,4)	35. (3)	36. (2)
37. (1)	38. (3)	39. (1,2)
40. (3,4)	41. (2,4)	42. (2,4)
43. (4)	44. (3)	45. (4)
46. (3)	47. (1,3)	48. (1,2,3,4)
49. (1,2,3)	50. (1,3)	51. (2)
52. (1)	53. (4)	54. (3)
55. (1,3)	56. (1,2,3,4)	57. (1,2,4)
58. (1,3)	59. (3)	60. (1)
61. (2)	62. (4)	63. (1,2)
64. (1,3)	65. (2,4)	66. (1)
67. (1)	68. (2)	69. (3)
70. (1,4)	71. (1,2,3,4)	72. (2,4)
73. (1,3)	74. (4)	75. (3)
76. (3)	77. (3)	78. (1)
79. (1,2,4)	80. (1,2,3,4)	81. (1,3,4)
82. (2)	83. (2)	84. (1)
85. (1)	86. (4)	87. (2)
88. (1,2)	89. (3)	90. (2)
91. (2)	92. (4)	93. (2)
94. (3)	95. (2,3)	96. (1,2)
97. (2,3)	98. (2)	99. (4)
100. (4)	101. (4)	102. (1,3)
103. (1,3)	104. (1,2,3,4)	105.(1,2,3)
106. (4)	107. (4)	108. (1)
109. (3)	110. (3)	111. (4)
112. (2,4)	113. (1,2,3)	114. (3)
115. (3)	116. (1)	117. (3)
118. (1,3)	119. (1,3,4)	120. (4)
121. (2)		

Phone: 9872311001 e–mail: mathsmim@gmail.com web: www.onlinemim.com