DIFFERENTIAL EQUATIONS

DECEMBER – 2014

PART – B

1. For $\lambda \in \mathbb{R}$, consider the boundary value problem

$$\left.\begin{array}{l} x^2\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}+\lambda y=0,\ x\in[1,2] \\ y(1)=y(2)=0 \end{array}\right\} -(P_\lambda)$$

Which of the following statement is true?

1. There exists a $\lambda_0 \in \mathbb{R}$ such that (P_λ) has a nontrivial solution for any $\lambda > \lambda_0$.
2. $\{\lambda \in \mathbb{R} : (P_\lambda)$ has a nontrivial solution$\}$ is a dense subset of $\mathbb{R}$.
3. For any continuous function $f:[1,2]\to \mathbb{R}$ with $f(x)\neq 0$ for some $x\in[1,2]$, there exists a solution u of (P_λ) for some $\lambda \in \mathbb{R}$ such that $\int_1^2 fu \neq 0$.
4. There exists a $\lambda \in \mathbb{R}$ such that (P_λ) has two linearly independent solutions.

2. Let y: $\mathbb{R} \to \mathbb{R}$ be differentiable and satisfy the ODE:

$$\left.\begin{array}{l} \dfrac{dy}{dx}=f(y),x\in\mathbb{R} \\ y(0)=y(1)=0 \end{array}\right\}$$

where f:$\mathbb{R}\to\mathbb{R}$ is a Lipschitz continuous function. Then

1. $y(x)=0$ if and only if $x\in\{0,1\}$
2. y is bounded
3. y is strictly increasing
4. dy/dx is unbounded

3. The system of ODE

$$\left.\begin{array}{l} \dfrac{dx}{dt}=(1+x^2)y,t\in\mathbb{R} \\ \dfrac{dy}{dt}=-(1+x^2)x,t\in\mathbb{R} \\ (x(0),y(0))=(a,b) \end{array}\right\}$$

has a solution:

1. only if (a,b)=(0,0)
2. for any $(a,b)\in\mathbb{R}$ x $\mathbb{R}$
3. such that $x^2(t)+y^2(t)=a^2+b^2$ for all $t\in\mathbb{R}$
4. such that $x^2(t)+y^2(t)\to\infty$ as $t\to\infty$ if a > 0 and b > 0

PART – C

4. Let y : $\mathbb{R}\to\mathbb{R}$ be a solution of the ODE

$$\left.\begin{array}{l} \dfrac{d^2y}{dx^2}-y=e^{-x},x\in\mathbb{R} \\ y(0)=\dfrac{dy}{dx}(0)=0 \end{array}\right\}$$

then

1. y attains its minimum on $\mathbb{R}$
2. y is bounded on $\mathbb{R}$
3. $\lim_{x\to\infty} e^{-x}y(x)=\dfrac{1}{4}$
4. $\lim_{x\to-\infty} e^{x}y(x)=\dfrac{1}{4}$

5. Let P,Q be continuous real valued functions defined on [-1,1] and u_i:[-1,1]→$\mathbb{R}$, $i=1,2$ be solutions of the ODE:

$$\frac{d^2u}{dx^2}+P(x)\frac{du}{dx}+Q(x)u=0,x\in[-1,1]$$

satisfying $u_1\geq 0, u_2\leq 0$ and $u_1(0)=u_2(0)=0$. Let W denote the Wronskian of u_1 and u_2, then

1. u_1 and u_2 are linearly independent
2. u_1 and u_2 are linearly dependent
3. $W(x)=0$ for all $x\in[-1,1]$
4. $W(x)\neq 0$ for some $x\in[-1,1]$

6. Let u(x,t)= $e^{i\omega x}v(t)$ with v(0)=1 be a solution to $\dfrac{\partial u}{\partial t}=\dfrac{\partial^3 u}{\partial x^3}$. Then

1. $u(x,t)=e^{i\omega(x-\omega^2 t)}$
2. $u(x,t)=e^{i\omega x-\omega^2 t}$
3. $u(x,t)=e^{i\omega(x+\omega^2 t)}$
4. $u(x,t)=e^{i\omega^3(x-t)}$

7. The Charpit's equations for PDE $up^2+q^2+x+y=0,\ p=\dfrac{\partial u}{\partial x}, q=\dfrac{\partial u}{\partial y}$ are given by

1. $\dfrac{dx}{-1-p^3}=\dfrac{dy}{-1-qp^2}=\dfrac{du}{2p^2u+2q^2}=\dfrac{dp}{2pu}=\dfrac{dq}{2q}$
2. $\dfrac{dx}{2pu}=\dfrac{dy}{2q}=\dfrac{du}{2p^2u+2q^2}=\dfrac{dp}{-1-p^3}$

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$$= \frac{dq}{-1-qp^2}$$

3. $\frac{dx}{up^2} = \frac{dy}{q^2} = \frac{du}{0} = \frac{dp}{x} = \frac{dq}{y}$

4. $\frac{dx}{2q} = \frac{dy}{2pu} = \frac{du}{x+y} = \frac{dp}{p^2} = \frac{dq}{qp^2}$

8. Consider the Cauchy problem of finding u=u(x,t) such that $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$ for x∈ℝ, t> 0

$u(x,0) = u_0(x), x \in \mathbb{R}$

Which choice(s) of the following functions for u_0 yield a C^1 solution u(x,t) for all $x \in \mathbb{R}$ and t > 0.

1. $u_0(x) = \frac{1}{1+x^2}$ 2. $u_0(x) = x$
3. $u_0(x) = 1+x^2$ 4. $u_0(x) = 1+2x$

9. Let u(x,t) satisfy for x ∈ ℝ, t > 0

$\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + 2\frac{\partial^2 u}{\partial x^2} = 0$. A solution of the form $u = e^{ix}v(t)$ with v(0)=0 and $v'(0) = 1$

1. is necessarily bounded
2. satisfies $|u(x,t)| < e^t$
3. is necessarily unbounded
4. is oscillatory in x.

10. Let u=u(x,t) be the solution of the Cauchy problem

$\frac{\partial u}{\partial t} + \left(\frac{\partial u}{\partial x}\right)^2 = 1$ x∈ℝ,t>0, $u(x,0) = -x^2$; $x \in \mathbb{R}$

Then

1. $u(x,t)$ exists for all $x \in$ ℝ and t > 0
2. $|u(x,t)| \to \infty$ as $t \to t^*$ for some $t^* > 0$ and $x \neq 0$
3. $u(x,t) \le 0$ for all $x \in$ ℝ and for all $t < \frac{1}{4}$.
4. $u(x,t) > 0$ for some $x \in \mathbb{R}$ and 0 < t< 1/4

JUNE – 2015

PART – B

11. The initial value problem $y' = 2\sqrt{y}$, $y(0) = a$, has

1. a unique solution if a < 0
2. no solution if a > 0
3. infinitely many solutions if a =0
4. a unique solution if a ≥ 0

12. Let y(x) be a continuous solution of the initial value problem $y' + 2y = f(x)$, $y(0) = 0$, $where\ f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$. Then $y\left(\frac{3}{2}\right)$ is equal to

1. $\frac{\sinh(1)}{e^3}$ 2. $\frac{\cosh(1)}{e^3}$
3. $\frac{\sinh(1)}{e^2}$ 4. $\frac{\cosh(1)}{e^2}$

13. The singular integral of the ODE $(xy' - y)^2 = x^2(x^2 - y^2)$ is

1. $y = x\sin x$ 2. $y = x\sin\left(x + \frac{\pi}{4}\right)$
3. $y = x$ 4. $y = x + \frac{\pi}{4}$

14. The function $G(x,\zeta) = \begin{cases} a + b\log\zeta, & 0 < x \le \zeta \\ c + d\log x, & \zeta \le x \le 1 \end{cases}$ is a Green's function for xy'' + y' = 0, subject to y being bounded as x→0 and y(1) = y'(1), if

1. a = 1, b = 1, c = 1, d = 1
2. a = 1, b = 0, c = 1, d = 0
3. a = 0, b = 1, c = 0, d = 1
4. a = 0, b = 0, c = 0, d = 0

15. For the initial value problem

$\frac{dy}{dx} = y^2 + \cos^2 x, \quad x > 0\ ;\ y(0) = 0,$

The largest interval of existence of the solution predicted by Picard's thorem is:

1. [0,1] 2. [0,1/2]
3. [0,1/3] 4. [0,1/4]

16. Let P be a continuous function on ℝ and W be the Wronskian of two linearly independent solutions y_1 and y_2 of the ODE:

$\frac{d^2y}{dx^2} + (1+x^2)\frac{dy}{dx} + P(x)y = 0, \ x \in \mathbb{R}.$

Let W(1) = a, W(2) = b and W(3) = c, then

1. a < 0 and b > 0
2. a < b < c or a > b > c
3. $\frac{a}{|a|} = \frac{b}{|b|} = \frac{c}{|c|}$
4. 0 < a < b and b > c >0

17. Consider the initial value problem $\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0,\ u(0,y)=4e^{-2y}$.

Then the value of u(1,1) is

1. $4e^{-2}$
2. $4e^{2}$
3. $2e^{-4}$
4. $4e^{4}$

18. Let $a,b\in\mathbb{R}$ be such that $a^2+b^2\neq 0$. Then the Cauchy problem $a\frac{\partial u}{\partial x}+b\frac{\partial u}{\partial y}=1;\ x,y\in\mathbb{R}$

$u(x,y)=x \text{ on } ax+by=1$

1. has more than one solution if either a or b is zero
2. has no solution
3. has a unique solution
4. has infinitely many solutions

19. The critical point of the system

$\frac{dx}{dt}=-4x-y,\ \frac{dy}{dt}=x-2y$ is an

1. asymptotically stable node
2. unstable node
3. asymptotically stable spiral
4. unstable spiral

20. The second order partial differential equation $u_{xx}+xu_{yy}=0$ is

1. elliptic for $x > 0$
2. hyperbolic for $x > 0$
3. elliptic for $x < 0$
4. hyperbolic for $x < 0$

PART – C

21. Which of the following are complete integrals of the partial differential equation $pqx+yq^2=1$?

1. $z=\frac{x}{a}+\frac{ay}{x}+b$
2. $z=\frac{x}{b}+\frac{ay}{x}+b$
3. $z^2=4(ax+y)+b$
4. $(z-b)^2=4(ax+y)$

22. For an arbitrary continuously differentiable function f, which of the following is a general solution of $z(px-qy)=y^2-x^2$

1. $x^2+y^2+z^2=f(xy)$
2. $(x+y)^2+z^2=f(xy)$
3. $x^2+y^2+z^2=f(y-x)$
4. $x^2+y^2+z^2=f((x+y)^2+z^2)$

DEC- 2015

PART – B

23. The PDE $\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=x$, has

1. only one particular integral.
2. a particular integral which is linear in x and y.
3. a particular integral which is a quadratic polynomial in x and y.
4. more than one particular integral.

24. The solution of the initial value problem $(x-y)\frac{\partial u}{\partial x}+(y-x-u)\frac{\partial u}{\partial y}=u$, $u(x,0)=1$, satisfies

1. $u^2(x-y+u)+(y-x-u)=0$.
2. $u^2(x+y+u)+(y-x-u)=0$
3. $u^2(x-y+u)-(x+y+u)=0$.
4. $u^2(y-x+u)+(x+y-u)=0$

25. Let u(x,y) be the solution of the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, which tends to zero as $y\to\infty$ and has the value sin x when y = 0. Then

1. $u=\sum_{n=1}^{\infty}a_n\sin(nx+b_n)e^{-ny}$, where a_n are arbitrary and b_n are non-zero constants.
2. $u=\sum_{n=1}^{\infty}a_n\sin(nx+b_n)e^{-n^2y}$, where $a_1=1$ and a_n ($n>1$), b_n are non-zero constants.
3. $u=\sum_{n=1}^{\infty}a_n\sin(nx+b_n)e^{-ny}$, where $a_1=1$, $a_n=0$ for $n>1$ and $b_n=0$ for $n\geq 1$.
4. $u=\sum_{n=1}^{\infty}a_n\sin(nx+b_n)e^{-n^2y}$, where $b_n=0$ for $n\geq 0$ and a_n are all nonzero.

PART – C

26. Let u(x,t) satisfy the wave equation

$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2};x\in(0,2\pi),t>0$ for some $\omega\in\mathbb{R}$.

$u(x,0)=e^{i\omega x}$

Then

1. $u(x,t)=e^{i\omega x}e^{i\omega t}$
2. $u(x,t)=e^{i\omega x}e^{-i\omega t}$
3. $u(x,t)=e^{i\omega x}\left(\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right)$.
4. $u(x,t)=t+\frac{x^2}{2}$.

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27. A solution of the PDE

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2-u=0$ represents

1. an ellipse in the x-y plane.
2. an ellipsoid in the xyu space.
3. a parabola in the u-x plane.
4. A hyperbola in the u-y plane.

28. Consider the ODE on ℝ y′(x) = f(y(x)). If f is an even function and y is an odd function, then

1. –y(-x) is also a solution
2. y(-x) is also a solution.
3. –y(x) is also a solution.
4. y(x) y (-x) is also a solution.

29. Consider the system of ODE in $\mathbb{R}^2$, $\frac{dY}{dt}=AY,\ Y(0)=\begin{pmatrix}0\\1\end{pmatrix}, t>0$ where

$A=\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}$ and $Y(t)=\begin{pmatrix}y_1(t)\\ y_2(t)\end{pmatrix}$. Then

1. $y_1(t)$ and $y_2(t)$ are monotonically increasing for t > 0.
2. $y_1(t)$ and $y_2(t)$ are monotonically increasing for t > 1.
3. $y_1(t)$ and $y_2(t)$ are monotonically decreasing for t > 0.
4. $y_1(t)$ and $y_2(t)$ are monotonically decreasing for t > 1.

30. Consider the boundary value problem

$-u''(x) = \pi^2 u(x)$; $x \in (0, 1)$

u(0) = u(1) = 0.

If u and u′ are continuous on [0, 1], then

1. $u'^2(x) + \pi^2 u^2(x) = u'^2(0)$
2. $\int_0^1 u'^2(x)\,dx-\pi^2\int_0^1 u^2(x)\,dx=0$
3. $u'^2(x) + \pi^2 u^2(x) = 0$
4. $\int_0^1 u'^2(x)\,dx-\pi^2\int_0^1 u^2(x)\,dx=u'^2(0)$

31. Let u(t) be a continuously differentiable function taking non negative values for t > 0 and satisfying $u'(t) = 4u^{3/4}(t)$; u(0) = 0. Then

1. u(t) = 0.
2. $u(t) = t^4$.
3. $u(t)=\begin{cases}0 & for\ \ 0<t<1\\ (t-1)^4 & for\ \ t\ge 1.\end{cases}$
4. $u(t)=\begin{cases}0 & for\ \ 0<t<10\\ (t-10)^4 & for\ \ t\ge 10.\end{cases}$

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PART – B

32. Let y_1 and y_2 be two solutions of the problem

$$\left.\begin{aligned} y''(t)+ay'(t)+by(t)=0, t\in R\\ y(0)=0\end{aligned}\right\}$$

where a and b are real constants. Let w be the Wronskian of y_1 and y_2. Then

1. $w(t)=0, \forall t\in R$
2. $w(t)=c, \forall t\in R$ for some positive constant c
3. w is a non constant positive function
4. There exist $t_1, t_2 \in R$ such that $w(t_1)<0<w(t_2)$

33. For the Cauchy problem

$u_t - uu_x = 0, x\in R, t>0$

$u(x,0)=x, x\in R,$

which of the following statements is true?

1. The solution u exists for all t > 0
2. The solution u exist for $t<\frac{1}{2}$ and breaks down at $t=\frac{1}{2}$
3. The solution u exist for $t<1$ and break down at t=1
4. The solution u exist for $t<2$ and breaks down at t=2

34. Let $A=\begin{bmatrix}-2 & 1 & 0\\ 0 & -2 & 1\\ 0 & 0 & -2\end{bmatrix}, x(t)=\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}$ and

$|x(t)|=(x_1^2(t)+x_2^2(t)+x_3^2(t))^{1/2}$

Then any solution of the first order system of the ordinary differential equation

$$\left.\begin{aligned} x'(t)=Ax(t)\\ x(0)=x_0\end{aligned}\right\}$$

Satisfies

1. $\lim_{t\to\infty}|x(t)|=0$
2. $\lim_{t\to\infty}|x(t)|=\infty$
3. $\lim_{t\to\infty}|x(t)|=2$
4. $\lim_{t\to\infty}|x(t)|=12$

35. Let a, b, c, d be four differentiable functions defined on $\mathbb{R}^2$. Then the partial differential equation

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$$\left(a(x,y)\frac{\partial}{\partial x}+b(x,y)\frac{\partial}{\partial y}\right)\left(c(x,y)\frac{\partial}{\partial x}+d(x,y)\frac{\partial}{\partial y}\right)u=0$$

is

1. always hyperbolic 2. always parabolic
3. never parabolic 4. never elliptic

PART – C

36. Consider the Cauchy problem for the Eikonal equation $p^2+q^2=1; p=\frac{\partial u}{\partial x}, q=\frac{\partial u}{\partial y}$ u(x,y) = 0 on x + y = 1. (x,y)$\in\mathbb{R}^2$. Then

1. The Charpit's equations for the differential equation are $\frac{dx}{dt}=2p; \frac{dy}{dt}=2q; \frac{du}{dt}=2; \frac{dp}{dt}=-p; \frac{dq}{dt}=-q.$
2. The Charpit's equations for the differential equation are $\frac{dx}{dt}=2p; \frac{dy}{dt}=2q; \frac{du}{dt}=2; \frac{dp}{dt}=0; \frac{dq}{dt}=0.$
3. $u(1,\sqrt{2})=\sqrt{2}$
4. $u(1,\sqrt{2})=1.$

37. Let y : $\mathbb{R}\to\mathbb{R}$ be a solution of the ordinary differential equation, $2y''+3y'+y=e^{-3x}$, x$\in\mathbb{R}$ satisfying $\lim_{x\to\infty} e^{x}y(x)=0$. Then

1. $\lim_{x\to\infty} e^{2x}y(x)=0.$
2. $y(0)=\frac{1}{10}.$
3. y is a bounded function on $\mathbb{R}$.
4. y(1) = 0.

38. For $\lambda\in\mathbb{R}$, consider the differential equation y′(x) = λsin(x+y(x)), y(0) = 1. Then this initial value problem has :

1. no solution in any neighbourhood of 0.
2. a solution in $\mathbb{R}$ if |λ| < 1.
3. a solution in a neighbourhood of 0.
4. a solution in $\mathbb{R}$ only if | λ| > 1.

39. The problem

$$\left.\begin{aligned} -y''+(1+x)y=\lambda y,\ x\in(0,1) \\ y(0)=y(1)=0 \end{aligned}\right\}$$

has a non zero solution

1. for all λ < 0.
2. for all λ∈ [0,1].
3. for some λ∈ (2, ∞).
4. for a countable number of λ's.

40. Let u : $\mathbb{R}\times[0,\infty)\to\mathbb{R}$ be a solution of the initial value problem

$$\left.\begin{aligned} u_{tt}-u_{xx}=0,\ for\ (x,t)\in R\times(0,\infty) \\ u(x,0)=f(x), x\in R \\ u_t(x,0)=g(x), x\in R \end{aligned}\right\}$$

Suppose f(x) = g(x) = 0 for x ∉[0,1], then we always have

1. u(x,t) = 0 for all (x,t)∈ (-∞,0) × (0,∞).
2. u(x,t) = 0 for all (x,t) ∈ (1,∞) × (0,∞).
3. u(x,t) = 0 for all (x,t) satisfying x + t < 0.
4. u(x,t) = 0 for all (x,t) satisfying x – t > 1.

41. Let u be the solution of the boundary value problem $u_{xx}+u_{yy}=0$ for 0 < x, y < π
u(x,0) = 0 = u(x,π) for 0 ≤ x ≤ π
u(0,y)=0, u(π,y)=sin y + sin 2y for 0≤ y≤π
Then

1. $u\left(1,\frac{\pi}{2}\right)=(\sinh(\pi))^{-1}\sinh(1).$
2. $u\left(1,\frac{\pi}{2}\right)=(\sinh(\pi))^{-1}\sinh(\pi).$
3. $u\left(1,\frac{\pi}{4}\right)=(\sinh(\pi))^{-1}(\sinh(1))\frac{1}{\sqrt{2}}+(\sinh(2\pi))^{-1}\sinh(2).$
4. $u\left(1,\frac{\pi}{4}\right)=(\sinh(1))^{-1}(\sinh(\pi))\frac{1}{\sqrt{2}}+(\sinh(2))^{-1}\sinh(2\pi).$

DEC – 2016

PART – B

42. Let (x(t), y(t)) satisfy the system of ODEs

$$\frac{dx}{dt}=-x+ty$$
$$\frac{dy}{dt}=tx-y$$

If $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ are two solutions and $\Phi(t)=x_1(t)y_2(t)-x_2(t)y_1(t)$ then $\frac{d\Phi}{dt}$ is equal to

1. -2Φ 2. 2Φ
3. $-\Phi$ 4. $-\Phi$

43. The boundary value problem $x^2y''-2xy'+2y=0$, subject to the boundary conditions

$y(1) + \alpha y'(1) = 1$, $y(2) + \beta y'(2) = 2$ has a unique solution if
1. $\alpha = -1, \beta = 2$.
2. $\alpha = -1, \beta = -2$.
3. $\alpha = -2, \beta = 2$.
4. $\alpha = -3, \beta = \frac{2}{3}$.

44. The PDE $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y} = 0$ is
1. hyperbolic for x > 0, y < 0.
2. elliptic for x > 0, y < 0.
3. hyperbolic for x > 0, y > 0.
4. elliptic for x < 0, y > 0.

45. Let u(x, t) satisfy the initial boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; $x \in (0, 1), t > 0$
$u(x, 0) = \sin(\pi x)$; $x \in [0, 1]$
$u(0, t) = u(1, t) = 0, t > 0$
Then for $x \in (0, 1)$, $u\left(x, \frac{1}{\pi^2}\right)$ is equal to
1. e sin(πx).
2. $e^{-1}\sin(\pi x)$.
3. sin(πx).
4. sin(πx).

46. Let $x : [0,3\pi] \to \mathbb{R}$ be a nonzero solution of the ODE $x''(t) + e^{t^2}x(t) = 0$, for $t \in [0,3\pi]$. Then the cardinality of the set $\{t \in [0,3\pi] : x(t) = 0\}$ is
1. equal to 1
2. greater than or equal to 2
3. equal to 2
4. greater than or equal to 3

PART – C

47. Consider the initial value problem $y'(t) = f(y(t)),\ y(0) = a \in \mathbb{R}$ where f: $\mathbb{R} \to \mathbb{R}$. Which of the following statements are necessarily true?
1. There exists a continuous function f: $\mathbb{R} \to \mathbb{R}$ and $a \in \mathbb{R}$ such that the above problem does not have a solution in any neighbourhood of 0.
2. The problem has a unique solution for every $a \in \mathbb{R}$ when f is Lipschitz continuous.
3. When f is twice continuously differentiable, the maximal interval of existence for the above initial value problem is $\mathbb{R}$.
4. The maximal interval of existence for the above problem is $\mathbb{R}$ when f is bounded and continuously differentiable.

48. Let (x(t),y(t)) satisfy for t > 0
$$\frac{dx}{dt} = -x + y,\ \frac{dy}{dt} = -y,\ x(0) = y(0) = 1.$$
Then x(t) is equal to
1. $e^{-t} + ty(t)$
2. $y(t)$
3. $e^{-t}(1+t)$
4. $-y(t)$

49. Consider the wave equation for u(x,t)
$$\left.\begin{aligned} &\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, (x,t) \in \mathbb{R} \times (0,\infty) \\ &u(x,0) = f(x), x \in \mathbb{R} \\ &\frac{\partial u}{\partial t}(x,0) = g(x), x \in \mathbb{R} \end{aligned}\right\}$$
Let u_i be the solution of the above problem with $f=f_i$ and $g=g_i$ for i=1,2, where f_i: $\mathbb{R} \to \mathbb{R}$ and g_i: $\mathbb{R} \to \mathbb{R}$ are given C^2 fucntions satisfying $f_1(x)=f_2(x)$ and $g_1(x)=g_2(x)$, for every $x \in [-1,1]$. Which of the following statements are necessarily true?
1. $u_1(0,1) = u_2(0,1)$
2. $u_1(1,1) = u_2(1,1)$
3. $u_1\left(\frac{1}{2},\frac{1}{2}\right) = u_2\left(\frac{1}{2},\frac{1}{2}\right)$
4. $u_1(0,2) = u_2(0,2)$

50. Let u: $\mathbb{R}^2 \backslash \{(0,0)\} \to \mathbb{R}$ be a C^2 function satisfying $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, for all $(x,y) \neq (0,0)$. Suppose u is of the form $u(x,y) = f\left(\sqrt{x^2+y^2}\right)$, where f: $(0,\infty) \to \mathbb{R}$, is a non-constant function, then
1. $\lim_{x^2+y^2 \to 0} |u(x,y)| = \infty$
2. $\lim_{x^2+y^2 \to 0} |u(x,y)| = 0$
3. $\lim_{x^2+y^2 \to \infty} |u(x,y)| = \infty$
4. $\lim_{x^2+y^2 \to \infty} |u(x,y)| = 0$

51. The Cauchy problem $\left.\begin{aligned} &y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0 \\ &u = g \ \ on\ \ \Gamma \end{aligned}\right\}$ has a unique solution in a neighborhood of Γ for every differentiable function g: $\Gamma \to \mathbb{R}$ if
1. $\Gamma = \{(x,0): x>0\}$
2. $\Gamma = \{(x,y): x^2+y^2 = 1\}$
3. $\Gamma = \{(x,y): x+y=1, x> 1\}$
4. $\Gamma = \{(x,y): y=x^2, x>0\}$

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PART – B

52. Suppose $x:[0,\infty)\to[0,\infty)$ is continuous and $x(0)=0$. If $(x(t))^2 \le 2+\int_0^t x(s)\,ds,\ \forall t\ge 0,$ then which of the following is TRUE?

1. $x(\sqrt{2})\in[0,2]$
2. $x(\sqrt{2})\in\left[0,\frac{3}{\sqrt{2}}\right]$
3. $x(\sqrt{2})\in\left[\frac{5}{\sqrt{2}},\frac{7}{\sqrt{2}}\right]$
4. $x(\sqrt{2})\in[10,\infty)$

53. The solution of the partial differential equation $u_t - xu_x + 1 - u = 0,\quad x\in\mathbb{R},\quad t>0$ subject to $u(x,0)=g(x)$ is

1. $u(x,t)=1-e^{-t}(1-g(xe^t))$
2. $u(x,t)=1+e^{t}(1-g(xe^t))$
3. $u(x,t)=1-e^{-t}(1-g(xe^{-t}))$
4. $u(x,t)=e^{-t}(1-g(xe^t))$

54. Suppose $u\in C^2(\overline{B}), B$ is the unit ball in $\mathbb{R}^2$, satisfies $\Delta u = f$ in B

$$\alpha u+\frac{\partial u}{\partial \mathbf{n}}=g \qquad \text{on } \partial B,\quad \alpha>0,$$

where $\mathbf{n}$ is the unit outward normal to B. If a solution exists then

1. it is unique
2. there are exactly two solutions
3. there are exactly three solutions
4. there are infinitely many solutions

PART – C

55. Consider the solution of the ordinary differential equation $y'(t)=-y^3+y^2+2y$ subject to $y(0)=y_0\in(0,2)$. Then $\lim_{t\to\infty} y(t)$ belongs to

1. {-1,0}
2. {-1,2}
3. {0,2}
4. {0,+∞}

56. If the solution to $\begin{cases}\frac{dy}{dx}=y^2+x^2, & x>0\\ y(0)=2\end{cases}$ exists in the interval [0, L_0) and the maximal interval of existence of $\begin{cases}\frac{dz}{dx}=z^2, & x>0\\ z(0)=1\end{cases}$ is [0,L_1), then which of the following statements are correct?

1. $L_1=1,\ L_0>1$
2. $L_1=1,\ L_0\le 1$
3. $L_1<2,\ L_0\le 1$
4. $L_1>2,\ L_0<1$

57. Consider the partial differential equation $x\frac{\partial u}{\partial x}+yu\frac{\partial u}{\partial y}=-xy$ for $x>0$ subject to u=5 on xy =1. Then

1. $u(x,y)$ exists when xy≤19 and $u(x,y)=u(y,x)$ for $x>0, y>0$
2. $u(x,y)$ exists when xy≥19 and $u(x,y)=u(y,x)$ for $x>0, y>0$
3. u (1,11)=3, u (13,-1)=7
4. u (1,-1)=5, u (11,1)=-5

58. If a complex integral of the partial differential equation $x(p^2+q^2)=zp;\ p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$ passes through the curve x=0, $z^2=4y$, then the envelope of this family passing through x=1 and y=1 has

1. z=-2
2. z=2
3. z=$\sqrt{2+2\sqrt{2}}$
4. z= - $\sqrt{2+2\sqrt{2}}$

59. For a differential function $f:\mathbb{R}\to\mathbb{R}$ define the difference quotient $(D_x f)(h)=\frac{f(x+h)-f(x)}{h}; h>0$. Consider numbers of the form $\hat{h}=h(1+\in)$ for a fixed $\in>0$ and let

$e_1(h)=f'(x)-(D_x f)(h)$, $e_2(h)=(D_x f)(h)-(D_x f)(\hat{h})$,
$e(h)=e_1(h)+e_2(h)$.

If $f(x+\hat{h})=f(x+h)$, then

1. $e_1(h)\to 0$ as $h\to 0$
2. $e_2(h)\to 0$ as $h\to 0$
3. $e_2(h)\to \in f'(x)/(1+\in)$ as $h\to 0$
4. $e(h)\to 0$ as $h\to 0$

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PART – B

60. Consider the differential equation

$(x-1)y''+xy'+\frac{1}{x}y=0$. Then

1. x = 1 is the only singular point
2. x = 0 is the only singular point
3. both x = 0 and x = 1 are singular points
4. neither x = 0 nor x = 1 are singular points

61. Let D denote the unit disc given by $\{(x, y) \mid x^2 + y^2 \leq 1\}$ and let D^c be its complement in the plane. The partial differential equation

$(x^2-1)\frac{\partial^2 u}{\partial x^2}+2y\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2}=0$ is

1. parabolic for all $(x, y) \in D^c$
2. hyperbolic for all $(x, y) \in D$
3. hyperbolic for all $(x, y) \in D^c$
4. parabolic for all $(x, y) \in D$

62. The set of real numbers λ for which the boundary value problem $\frac{d^2y}{dx^2}+\lambda y=0$, $y(0)= 0$, $y(\pi) = 0$ has nontrivial solutions is

1. $(-\infty, 0)$
2. $\{\sqrt{n} \mid n$ is a positive integer$\}$
3. $\{n^2 \mid n$ is a positive integer$\}$
4. $\mathbb{R}$

63. Let u(x, t) be the solution of the initial value problem

$u_{tt} - u_{xx} = 0$, $u(x, 0) = x^3$, $u_t(x, 0) = \sin x$

Then $u(\pi, \pi)$ is

1. $4\pi^3$ 2. π^3 3. 0 4. 4

64. Let u(x, t) be a solution of the heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ in a rectangle $[0, \pi] \times [0, T]$ subject to the boundary conditions $u(0, t) = u(\pi,t) = 0$, $0 \leq t \leq T$ and the initial condition $u(x, 0) = \varphi(x)$, $0 \leq x \leq \pi$. If $f(x) = u(x, T)$, then which of the following is true for a suitable kernel k(x, y)?

1. $\int_0^{\pi} k(x,y)\varphi(y)\,dy = f(x),\ 0\leq x\leq \pi$
2. $\varphi(x)+\int_0^{\pi} k(x,y)\varphi(y)\,dy = f(x),\ \ 0\leq x\leq \pi$
3. $\int_0^{x} k(x,y)\varphi(y)\,dy = f(x),\ 0\leq x\leq \pi$
4. $\varphi(x)+\int_0^{x} k(x,y)\varphi(y)\,dy = f(x),\ \ 0\leq x\leq \pi$

PART – C

65. Consider a system of first order differential equations $\frac{d}{dt}\begin{bmatrix} x(t)\\ y(t)\end{bmatrix}=\begin{bmatrix} x(t)+y(t)\\ -y(t)\end{bmatrix}$.

The solution space is spanned by

1. $\begin{bmatrix} 0\\ e^{-t}\end{bmatrix}$ *and* $\begin{bmatrix} e^{t}\\ 0\end{bmatrix}$
2. $\begin{bmatrix} e^{t}\\ 0\end{bmatrix}$ *and* $\begin{bmatrix} \cosh t\\ e^{-t}\end{bmatrix}$
3. $\begin{bmatrix} e^{-t}\\ -2e^{-t}\end{bmatrix}$ *and* $\begin{bmatrix} \sinh t\\ e^{-t}\end{bmatrix}$
4. $\begin{bmatrix} e^{t}\\ 0\end{bmatrix}$ *and* $\begin{bmatrix} e^{t}-\frac{1}{2}e^{-t}\\ e^{-t}\end{bmatrix}$

66. Consider the differential equation $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}-y=0$ defined on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

Which among the following are true ?

1. there is exactly one solution y=y(x) with $y(0) = y'(0) = 1$ and $y\left(\frac{\pi}{3}\right)=2\left(1+\frac{\pi}{3}\right)$
2. there is exactly one solution y=y(x) with $y(0) = 1$, $y'(0) =- 1$ and $y\left(-\frac{\pi}{3}\right)=2\left(1+\frac{\pi}{3}\right)$
3. any solution y=y(x) satisfies $y''(0)=y(0)$
4. If y_1 and y_2 are any two solutions then $(ax+b)\, y_1=(cx+d)y_2$ for some $a,b,c,d\in\mathbb{R}$

67. Consider a boundary value problem (BVP) $\frac{d^2y}{dx^2}=f(x)$ with boundary conditions $y(0)=y(1)=y'(1)$, where f is a real-valued continuous function on [0,1]. Then which of the following are true ?

1. the given BVP has a unique solution for every f
2. the given BVP does not have a unique solution for some f
3. $y(x)=\int_0^x xtf(t)dt+\int_x^1 (t-x+xt)f(t)dt$ is a solution of the given BVP
4. $y(x)=\int_0^x (x-t+xt)f(t)dt+\int_x^t xtf(t)dt$ is a solution of the given BVP

68. Consider the Lagrange equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$. Then the general solution of the given equation is

1. $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ for an arbitrary differentiable function F
2. $F\left(\frac{x-y}{z}, \frac{1}{x} - \frac{1}{y}\right) = 0$ for an arbitrary differentiable function F
3. $z = f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f
4. $z = xyf\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f

69. Consider the second order PDE $8\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} - 3\frac{\partial^2 z}{\partial y^2} = 0$. Then which of the following are correct ?

1. the equation is elliptic
2. the equation is hyperbolic
3. the general solution is $z = f\left(y - \frac{x}{2}\right) + g\left(y + \frac{3x}{4}\right)$, for arbitrary differentiable functions f and g
4. the general solution is $z = f\left(y + \frac{x}{2}\right) + g\left(y - \frac{3x}{4}\right)$, for arbitrary differentiable functions f and g

70. Let $B = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 < 1\}$, and let $C_{1d}^2(\overline{B}; \mathbb{R}^2)$={u∈$C^2$ (B;$\mathbb{R}^2$)|u(x_1,x_2) = (x_1,x_2) for (x_1,x_2) ∈ ∂B}. Let u=(u_1,u_2) and define $J : C_{1d}^2(\overline{B}; \mathbb{R}^2) \to \mathbb{R}$ by

$$J(u) = \int_B \left(\frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_2} - \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1}\right) dx_1 dx_2$$ Then

1. $\inf\{J(u) : u \in C_{1d}^2(\overline{B}; \mathbb{R}^2)\}$=0
2. $J(u) > 0$, for all $u \in C_{1d}^2(\overline{B}; \mathbb{R}^2)$
3. f(u)=1, for infinitely many $u \in C_{1d}^2(B; \mathbb{R}^2)$
4. J (u)=π, for all $u \in C_{1d}^2(\overline{B}; \mathbb{R}^2)$

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PART-B

71. Consider the ordinary differential equation $y' = y(y-1)(y-2)$. Which of the following statements is true ?
1. If y(0) = 0.5 then y is decreasing
2. If y(0) = 1.2 then y is increasing
3. If y(0) = 2.5 then y is unbounded
4. If y(0) < 0 then y is bounded below

72. Consider the ordinary differential equation $y'' + P(x)y' + Q(x)y = 0$ where P and Q are smooth functions. Let y_1 and y_2 be any two solutions of the ODE. Let W(x) be the corresponding Wronskian. Then which of the following is always true?

1. If y_1 and y_2 are linearly dependent then ∃ x_1, x_2 such that W(x_1) = 0 and W(x_2) ≠ 0
2. If y_1 and y_2 are linearly independent then W(x) = 0 ∀ x
3. If y_1 and y_2 are linearly dependent then W(x) ≠ 0 ∀ x
4. If y_1 and y_2 are linearly independent then W(x) ≠ 0 ∀ x

73. The Cauchy problem

$$\left.\begin{array}{l} 2u_x + 3u_y = 5 \\ u = 1 \ \ on \ \ the \ \ line \ \ 3x - 2y = 0 \end{array}\right\}$$ has

1. exactly one solution
2. exactly two solutions
3. infinitely many solutions
4. no solution

74. Let u be the unique solution of

$$\left.\begin{array}{l} \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, x \in \mathbb{R}, \ t>0 \\ u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = 0, x \in \mathbb{R} \end{array}\right\}$$

where f: $\mathbb{R} \to \mathbb{R}$ satisfies the relations f(x) = x(1-x) ∀ x∈[0,1] and f(x+1)= f(x) ∀ x∈$\mathbb{R}$. Then $u\left(\frac{1}{2}, \frac{5}{4}\right)$ is

1. $\frac{1}{8}$
2. $\frac{1}{16}$
3. $\frac{3}{16}$
4. $\frac{5}{16}$

PART-C

75. Let a be a fixed real constant. Consider the first order partial differential equation $\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=0,$ $x \in \mathbb{R}$, $t > 0$ with the initial data $u(x, 0) = u_0(x)$, $x \in \mathbb{R}$ where u_0 is a continuously differentiable function. Consider the following two statements

S_1 : There exists a bounded function u_0 for which the solution u is unbounded

S_2 : If u_0 vanishes outside a compact set then for each fixed $T > 0$ there exists a compact set $K_T \subset \mathbb{R}$ such that $u(x, T)$ vanishes for $x \notin K_T$.

Which of the following are true?
1. S_1 is true and S_2 is false
2. S_1 is true and S_2 is also true
3. S_1 is false and S_2 is true
4. S_1 is false and S_2 is also false

76. If u(x,t) is the solution of $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, $0<x<1$, $t>0$
$u(x, 0) = 1 + x + \sin(\pi x)\cos(\pi x)$
$u(0, t) = 1$, $u(1, t) = 2$.
then
1. $u\left(\frac{1}{2},\frac{1}{4}\right)=\frac{3}{2}$
2. $u\left(\frac{1}{2},\frac{1}{2}\right)=\frac{3}{2}$
3. $u\left(\frac{1}{4},\frac{3}{4}\right)=\frac{5}{4}+\frac{1}{2}e^{-3\pi^2}$
4. $u\left(\frac{1}{4},1\right)=\frac{5}{4}+\frac{1}{2}e^{-4\pi^2}$

77. Assume that $a : [0, \infty) \to \mathbb{R}$ is a continuous function. Consider the ordinary differential equation $y'(x) = a(x)\, y(x)$, $x > 0$, $y(0) = y_0 \neq 0$.
Which of the following statements are true?
1. If $\int_0^\infty |a(x)|\,dx<\infty$, then y is bounded
2. If $\int_0^\infty |a(x)|\,dx<\infty$, then $\lim_{x\to\infty} y(x)$ exists
3. If $\lim_{x\to\infty} a(x) = 1$, then $\lim_{x\to\infty}|y(x)| = \infty$
4. If $\lim_{x\to\infty} a(x) = 1$, then y is monotone

78. Consider the system of differential equations
$$\frac{dx}{dt}=2x-7y$$
$$\frac{dy}{dt}=3x-8y$$
Then the critical point (0, 0) of the system is an
1. asymptotically stable node
2. unstable node
3. asymptotically stable spiral
4. unstable spiral

79. Consider the Sturm-Liouville problem $y'' + \lambda y = 0$, $y(0) = 0$ and $y(\pi) = 0$.Which of the following statements are true?
1. There exist only countably many characteristic values
2. There exist uncountably many characteristic values
3. Each characteristic function corresponding to the characteristic value λ has exactly $[\sqrt{\lambda}]-1$ zeroes in $(0, \pi)$
4. Each characteristic function corresponding to the characteristic value λ has exactly $[\sqrt{\lambda}]$ zeroes in $(0, \pi)$

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PART-B

80. Let u(x,t) be a function that satisfies the PDE $u_{xx}-u_{tt}=e^x+6t, x\in\mathbb{R}$, $t > 0$ and the initial conditions $u(x,0) = \sin(x)$, $u_t(x,0) = 0$ for every $x\in\mathbb{R}$. Here subscripts denote partial derivatives corresponding to the variables indicated. Then the value of $u\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ is
1. $e^{\pi/2}\left(1+\frac{1}{2}e^{\pi/2}\right)+\left(\frac{\pi^3+4}{8}\right)$
2. $e^{\pi/2}\left(1+\frac{1}{2}e^{\pi/2}\right)+\left(\frac{\pi^3-4}{8}\right)$
3. $e^{\pi/2}\left(1-\frac{1}{2}e^{\pi/2}\right)-\left(\frac{\pi^3+4}{8}\right)$
4. $e^{\pi/2}\left(1-\frac{1}{2}e^{\pi/2}\right)-\left(\frac{\pi^3-4}{8}\right)$

81. Let u(x,t) satisfy the IVP: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, x\in\mathbb{R}$, $t>0$, $u(x,0)=\begin{cases}1, & 0\le x\le 1\\ 0, & elsewhere.\end{cases}$ Then the value of $\lim_{t\to 0^+} u(1,t)$ equals

1. e 2. π
3. 1/2 4. 1

82. If $y_1(x)$ and $y_2(x)$ are two solutions of the Differential equation $(\cos x)y''+(\sin x)y'-(1+e^{-x^2})y=0\ \forall\ x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ with $y_1(0)=\sqrt{2}, y_1'(0)=1, y_2(0)=-\sqrt{2}, y_2'(0)=2,$ then the Wronskian of $y_1(x)$ and $y_2(x)$ at $x=\frac{\pi}{4}$ is

1. $3\sqrt{2}$
2. 6
3. 3
4. $-3\sqrt{2}$

83. The critical point (0,0) for the system
$$x'(t)=x-2y+y^2\sin x$$
$$y'(t)=2x-2y-3y\cos(y^2)$$
is a

1. Stable spiral point
2. Unstable spiral point
3. Saddle point
4. Stable node

PART-C

84. Let u(x, t) be a function that satisfies the PDE : $u_t + uu_x = 1$, $x \in \mathbb{R}$, $t > 0$, and the initial condition $u\left(\frac{t^2}{4},t\right)=\frac{t}{2}$. Then the IVP has

1. only one solution
2. two solutions
3. an infinite number of solutions
4. solutions none of which is differentiable on the characteristic base curve

85. Let u(x) satisfy the boundary value problem
$$(BVP)\begin{cases} u''+u'=0, & x\in(0,1)\\ u(0)=0 \\ u(1)=1\end{cases}$$
Consider the finite difference approximation to (BVP)
$$(BVP)_h\begin{cases}\dfrac{U_{j+1}-2U_j+U_{j-1}}{h^2}+\dfrac{U_{j+1}-U_{j-1}}{2h}=0,\ j=1,\ldots,N-1\\ U_0=0\\ U_N=1\end{cases}$$

Here U_j is an approximation to $u(x_j)$ where $x_j = jh$, j = 0, …,N is a partition of [0, 1] with h = 1/N for some positive integer N. Then which of the following are true?

1. There exists a solution to $(BVP)_h$ of the form $U_j = ar^j + b$ for some $a, b \in \mathbb{R}$ with $r\neq1$ and r satisfying $(2+h)r^2 - 4r + (2-h)=0$
2. $U_j = (r^j - 1) / (r^N - 1)$ where r satisfies $(2 + h) r^2 - 4r + (2 - h) = 0$ and $r \neq 1$
3. u is monotonic in x
4. U_j is monotonic in j.

86. Three solutions of a certain second order non-homogenous linear differential equation are $y_1(x)=1+xe^{x^2}, y_2(x)=(1+x)e^{x^2}+1, y_3(x)=1+e^{x^2}$.
Which of the following is (are) general solution(s) of the differential equation?

1. $(C_1 + 1)y_1 + (C_2 - C_1)y_2 - C_2y_3$, where C_1 and C_2 are arbitrary constants
2. $C_1 (y_1 - y_2) + C_2 (y_2 - y_3)$, where C_1 and C_2 are arbitrary constants
3. $C_1 (y_1 - y_2) + C_2 (y_2 - y_3) + C_3 (y_3 - y_1)$, where C_1, C_2 and C_3 are arbitrary constants
4. $C_1 (y_1 - y_3) + C_2 (y_3 - y_2) + y_1$, where C_1 and C_2 are arbitrary constants

87. The method of variation of parameters to solve the differential equation $y'' + p(x)y' + q(x)y\ r(x)$, where $x \in I$ and p(x), q(x), r(x) are non-zero continuous functions on an interval I, seeks a particular solution of the form $y_p(x) = v_1(x)y_1(x) + v_2(x)y_2(x)$, where y_1 and y_2 are linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$ and $v_1(x)$ and $v_2(x)$ are functions to be determined. Which of the following statements are necessarily true?

1. The Wronskian of y_1 and y_2 is never zero in I.
2. v_1, v_2 and $v_1y_1 + v_2y_2$ are twice differentiable
3. v_1 and v_2 may not be twice differentiable, but $v_1y_1 + v_2y_2$ is twice differentiable
4. The solution set of $y'' + p(x)y' + q(x)y = r(x)$ consists of functions of the form $ay_1 + by_2 + y_p$ where $a, b \in\mathbb{R}$ are arbitrary constants

88. Consider the eigenvalue problem $y'' + \lambda y = 0$ for $x \in (-1, 1)$, $y(-1) = y(1)$, $y'(-1) = y'(1)$. Which of the following statements are true?

1. All eigenvalues are strictly positive
2. All eigenvalues are non-negative

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3. Distinct eigenfunctions are orthogonal in $L^2[-1, 1]$.
4. The sequence of eigenvalues is bounded above

89. Consider the IVP: $xu_x + tu_t = u + 1$, $x \in \mathbb{R}$, $t \geq 0$
$u(x, t) = x^2$, $t = x^2$. Then
1. the solution is singular at (0, 0)
2. the given space curve $(x, t, u) = (\xi, \xi^2, \xi^2)$ is not a characteristic curve at (0, 0)
3. there is no base-characteristic curve in the (x, t) plane passing through (0,0).
4. a necessary condition for the IVP to have a unique C^1 solution at (0, 0) does not hold

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PART – B

90. Let y(x) be the solution of $x^2y''(x) - 2y(x)=0$, $y(1) = 1$, $y(2) = 1$. Then the value of y(3) is
1. $\frac{11}{21}$
2. 1
3. $\frac{17}{21}$
4. $\frac{11}{7}$

91. The positive values of λ for which the equation $y''(x) + \lambda^2 y(x) = 0$ has non-trivial solution satisfying $y(0) = y(\pi)$ and $y'(0) = y'(\pi)$ are
1. $\lambda = \frac{2n+1}{2}, n = 1, 2, \ldots$
2. $\lambda = 2n$, n = 1, 2, …
3. $\lambda = n$, n = 1, 2, …
4. $\lambda = 2n - 1$, n = 1, 2, …

92. Consider the PDE

$$P(x,y)\frac{\partial^2 u}{\partial x^2} + e^{x^2} e^{y^2} \frac{\partial^2 u}{\partial x \partial y} + Q(x,y)\frac{\partial^2 u}{\partial y^2} + e^{2x}\frac{\partial u}{\partial x} + e^{y}\frac{\partial u}{\partial y} = 0,$$

where P and Q are polynomials in two variables with real coefficients. Then which of the following is true for all choices of P and Q?
1. There exists R > 0 such that the PDE is elliptic in $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > R\}$
2. There exists R > 0 such that the PDE is hyperbolic in $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > R\}$
3. There exists R > 0 such that PDE is parabolic in $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > R\}$
4. There exists R > 0 such that the PDE is hyperbolic in $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < R\}$

93. Let u be the unique solution of

$$\left.\begin{array}{ll} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} \text{ where } (x,t) \in (0,1)\times(0,\infty) & \\ u(x,0) = \sin \pi x, & x \in (0,1) \\ u(0,t) = u(1,t) = 0, & t \in (0,\infty) \end{array}\right\}$$

Then which of the following is true?
1. There exists $(x, t) \in (0, 1) \times (0, \infty)$ such that u(x, t) = 0
2. There exists $(x, t) \in (0, 1) \times (0, \infty)$ such that $\frac{\partial u}{\partial t}(x,t) = 0$
3. The function $e^t u(x, t)$ is bounded for $(x, t) \in (0, 1) \times (0, \infty)$
4. There exists $(x, t) \in (0, 1) \times (0, \infty)$ such that u(x, t) > 1

PART – C

94. Let $y_1(x)$ be any non-trivial real valued solution of $y''(x) + xy(x) = 0$, $0 < x < \infty$. Let $y_2(x)$ be the solution of $y''(x) + y(x) = x^2 + 2$, $y(0) = y'(0) = 0$. Then
1. $y_1(x)$ has infinitely many zeros.
2. $y_2(x)$ has infinitely many zeros
3. $y_1(x)$ has finitely many zeros
4. $y_2(x)$ has finitely many zeros

95. Consider the equation $y''(x) + a(x)\, y(x) = 0$, a(x) is continuous function with period T. Let $\phi_1(x)$ and $\phi_2(x)$ be the basis for the solution satisfying $\phi_1(0) = 1$, $\phi_1'(0) = 0$, $\phi_2(0) = 0$, $\phi_2'(0) = 1$. Let W (ϕ_1, ϕ_2) denote the Wronskian of ϕ_1 and ϕ_2. Then
1. $W(\phi_1, \phi_2) = 1$
2. $W(\phi_1, \phi_2) = e^x$
3. $\phi_1(T) + \phi_2'(T) = 2$ if the given differential equation has a nontrivial periodic solution with period T
4. $\phi_1(T) + \phi_2'(T) = 1$ if the given differential equation has a nontrivial periodic solution with period T

96. Let $f : \mathbb{R} \to \mathbb{R}$ be a Lipschitz function such that f(x) = 0 if and only if $x = \pm n^2$ where

$n \in \mathbb{N}$. Consider the initial value problem: $y'(t) = f(y(t))$, $y(0) = y_0$.
Then which of the following are true?

1. y is a monotone function for all $y_0 \in \mathbb{R}$
2. for any $y_0 \in \mathbb{R}$, there exists $M_{y_0} > 0$ such that $|y(t)| \le M_{y_0}$ for all $t \in \mathbb{R}$
3. there exists a $y_0 \in \mathbb{R}$, such that the corresponding solution y is unbounded
4. $\sup_{t,s\in\mathbb{R}} |y(t) - y(s)| = 2n + 1$ if $y_0 \in (n^2, (n + 1)^2)$, $n \ge 1$

97. The general solution $z = z(x, y)$ of $(x+y)z\, z_x + (x - y)z\, z_y = x^2 + y^2$ is

1. $F(x^2 + y^2 + z^2, z^2 - xy) = 0$ for arbitrary C^1 function F
2. $F(x^2 - y^2 - z^2, z^2 - 2xy) = 0$ for arbitrary C^1 function F
3. $F(x + y + z, z - 2xy) = 0$ for arbitrary C^1 function F
4. $F(x^3 - y^3 - z^3, z - 2x^2y^2) = 0$ for arbitrary C^1 function F

98. Let u be the solution of the problem

$$\left.\begin{aligned}&\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, && (x,y)\in(0,\pi)\times(0,\pi),\\ &u(0,y)=u(\pi,y)=0, && y\in(0,\pi),\\ &u(x,0)=0, u(x,\pi)=\sin(2x), && x\in(0,\pi).\end{aligned}\right\}$$

Then

1. $\max\{u(x, y) : 0 \le x, y \le \pi\} = 1$
2. $u(x_0, y_0) = 1$ for some $(x_0, y_0) \in (0, \pi) \times (0, \pi)$
3. $u(x, y) > -1$ for all $(x, y) \in (0, \pi) \times (0, \pi)$
4. $\min\{u(x, y) : 0 \le x, y \le \pi\} > -1$

99. Let u be the solution of

$$\left.\begin{aligned}&\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}, && (x,t)\in\mathbb{R}\times(0,\infty),\\ &u(x,0)=f(x), && x\in\mathbb{R},\\ &u_t(x,0)=g(x), && x\in\mathbb{R}\end{aligned}\right\}$$

where f, g are in $C^2(\mathbb{R})$ and satisfy the following conditions
(i) $f(x) = g(x) = 0$ for $x \le 0$,
(ii) $0 < f(x) \le 1$ for $x > 0$,
(iii) $g(x) > 0$ for $x > 0$
(iv) $\int_0^\infty g(x)\,dx < \infty$.
Then, which of the following statements are true?

1. $u(x, t) = 0$ for all $x \le 0$ and $t > 0$
2. u is bounded on $\mathbb{R} \times (0, \infty)$
3. $u(x, t) = 0$ whenever $x + t < 0$
4. $u(x, t) = 0$ for some (x, t) satisfying $x + t > 0$

DECEMBER-2019

PART-B

100. Let u(x, y) be the solution of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=64$ in the unit disc $\{(x, y) \mid x^2 + y^2 < 1\}$ and such that u vanishes on the boundary of the disc. Then $u\left(\frac{1}{4},\frac{1}{\sqrt{2}}\right)$ is equal to

1. 7   2. 16
3. -7   4. -16

101. For the following system of ordinary differential equations

$$\frac{dx}{dt}=x(3-2x-2y),$$
$$\frac{dy}{dt}=y(2-2x-y),$$

the critical point (0, 2) is

1. a stable spiral
2. an unstable spiral
3. a stable node
4. an unstable node

102. The Cauchy problem

$$y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}=0$$

and $x_0(s) = \cos(s)$, $y_0(s) = \sin(s)$, $z_0(s) = 1$, $s > 0$ has

1. a unique solution
2. no solution
3. more than one but finite number of solutions
4. infinitely many solutions

103. Consider the system of ordinary differential equations

$$\frac{dx}{dt}=4x^3y^2-x^5y^4,$$
$$\frac{dy}{dt}=x^4y^5+2x^2y^3.$$

Then for this system there exists

1. a closed path in $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \le 5\}$
2. a closed path in $\{(x,y)\in\mathbb{R}^2 \mid 5<x^2+y^2\le 10\}$

3. a closed path in $\{(x,y)\in\mathbb{R}^2 \mid x^2 + y^2 > 10\}$
4. no closed path in $\mathbb{R}^2$

PART – C

104. Consider the initial value problem $\frac{dy}{dx} = x^2 + y^2$, y(0) = 1; $0 \le x \le 1$. Then which of the following statements are true?
1. There exists a unique solution in $\left[0, \frac{\pi}{4}\right]$
2. Every solution is bounded in $\left[0, \frac{\pi}{4}\right]$
3. The solution exhibits a singularity at some point in [0, 1]
4. The solution becomes unbounded in some subinterval of $\left[\frac{\pi}{4}, 1\right]$

105. Let u(x, t) be the solution of

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = xt,\ -\infty < x < \infty, t > 0,$$

$$u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0, -\infty < x < \infty.$$

Then u(2, 3) is equal to
1. 9 2. 1
3. 27 4. 12

106. Consider the eigenvalue problem
$((1 + x^4)y')' + \lambda y = 0$, $x \in (0, 1)$,
$y(0) = 0$, $y(1) + 2y'(1) = 0$.
Then which of the following statements are true?
1. all the eigenvalues are negative
2. all the eigenvalues are positive
3. there exist some negative eigenvalues and some positive eigenvalues
4. there are no eigenvalues

107. Let y be a solution of
$(1 + x^2)y'' + (1 + 4x^2)y = 0$, $x > 0$
y(0) = 0. Then y has
1. infinitely many zeroes in [0, 1]
2. infinitely many zeroes in $[1, \infty)$
3. at least n zeroes in $[0, n\pi]$, $\forall\, n \in \mathbb{N}$
4. at most 3n zeroes in $[0, n\pi]$, $\forall\, n \in \mathbb{N}$

108. A possible initial strip $(x_0, y_0, z_0, p_0, q_0)$ for the Cauchy problem pq = 1 where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ and $x_0(s)=s$, $y_0(s)= \frac{1}{s}$, $z_0(s) = 1$ for s > 1 is
1. $\left(s, \frac{1}{s}, 1, \frac{1}{s}, s\right)$
2. $\left(s, \frac{1}{s}, 1, -\frac{1}{s}, -s\right)$
3. $\left(s, \frac{1}{s}, 1, \frac{1}{s}, -s\right)$
4. $\left(s, \frac{1}{s}, 1, -\frac{1}{s}, s\right)$

JUNE-2020

PART-B

109. Let k be a positive integer. Consider the differential equation

$$\begin{cases} \frac{dy}{dt} = y^{\frac{5k}{5k+2}} & for\ t > 0, \\ y(0) = 0 \end{cases}$$

Which of the following statements is true?
1. It has a unique solution which is continuously differentiable on $(0, \infty)$
2. It has at most two solutions which are continuously differentiable on $(0, \infty)$
3. It has infinitely many solutions which are continuously differentiable on $(0, \infty)$
4. It has no continuously differentiable solution on $(0, \infty)$

110. Let $y_0 > 0$, $z_0 > 0$ and $\alpha > 1$.
Consider the following two different equations:

$$(*)\ \begin{cases} \frac{dy}{dt} = y^{\alpha} & for\, t > 0, \\ y(0) = y_0 \end{cases}$$

$$(**)\begin{cases} \frac{dz}{dt} = -z^{\alpha} & for\, t > 0, \\ z(0) = z_0 \end{cases}$$

We say that the solution to a differential equation exists globally if it exists for all t > 0.
Which of the following statements is true?
1. Both $(*)$ and $(**)$ have global solutions
2. None of $(*)$ and $(**)$ have global solutions

3. There exists a global solution for ($*$) and there exists a T < ∞ such that $\lim_{t\to T}|z(t)| = +\infty$
4. There exists a global solution for ($**$) and there exists a T < ∞ such that $\lim_{t\to T}|y(t)| = +\infty$

111. The general solution of the surfaces which are perpendicular to the family of surfaces $z^2 = kxy$, $k \in \mathbb{R}$ is

1. $\phi(x^2 - y^2, xz) = 0$, $\phi \in C^1(\mathbb{R}^2)$
2. $\phi(x^2 - y^2, x^2 + z^2) = 0$, $\phi \in C^1(\mathbb{R}^2)$
3. $\phi(x^2 - y^2, 2x^2 + z^2) = 0$, $\phi \in C^1(\mathbb{R}^2)$
4. $\phi(x^2 + y^2, 3x^2 - z^2) = 0$, $\phi \in C^1(\mathbb{R}^2)$

112. The general solution of the equation

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$

is

1. $z = \varphi\left(\frac{|x|}{|y|}\right), \varphi \in C^1(\mathbb{R})$
2. $z = \varphi\left(\frac{x-1}{y}\right), \varphi \in C^1(\mathbb{R})$
3. $z = \varphi\left(\frac{x+1}{y}\right), \varphi \in C^1(\mathbb{R})$
4. $z = \varphi(|x| + |y|), \varphi \in C^1(\mathbb{R})$

PART – C

113. The following two-point boundary value problem

$$\begin{cases} y''(x) + \lambda y(x) = 0 \text{ for } x \in (0,\pi) \\ y(0) = 0 \\ y(\pi) = 0 \end{cases}$$

has a trivial solution y = 0. It also has a non-trivial solution for

1. no values of $\lambda \in \mathbb{R}$
2. $\lambda = 1$
3. $\lambda = n^2$ for all $n \in \mathbb{N}$, $n > 1$
4. $\lambda \le 0$

114. Let A be an n × n matrix with distinct eigenvalues $(\lambda_1, \ldots, \lambda_n)$ with corresponding linearly independent eigenvectors $(v_1, \ldots, v_n)$.

Then, the non-homogenous differential equation

$$x'(t) = Ax(t) + e^{\lambda_1 t} v_1$$

1. does not have a solution of the form $e^{\lambda_1 t}a$ for any vector $a \in \mathbb{R}^n$
2. has a solution of the form $e^{\lambda_1 t}a$ for some vector $a \in \mathbb{R}^n$
3. has a solution of the form $e^{\lambda_1 t}a + te^{\lambda_1 t}b$ for some vectors a, $b \in \mathbb{R}^n$
4. does not have a solution of the form $e^{\lambda_1 t}a + te^{\lambda_1 t}b$ for any vectors a, $b \in \mathbb{R}^n$

115. Consider the solutions

$$y_1 := \begin{pmatrix} e^{-3t} \\ e^{-3t} \\ 0 \end{pmatrix} \text{ and } y_2 := \begin{pmatrix} 0 \\ e^{-5t} \\ e^{-5t} \end{pmatrix}$$

be the homogenous linear system of differential equation

$$(*)\ y'(t) = \begin{pmatrix} -5 & 2 & -2 \\ 1 & -4 & -1 \\ -1 & 1 & -6 \end{pmatrix} y(t).$$

Which of the following statements are true?

1. y_1 and y_2 form a basis for the set of all solutions to ($*$)
2. y_1 and y_2 are linearly independent but do not form a basis for the set of all solutions to ($*$)
3. There exists another solution y_1 such that (y_1, y_2, y_3) form a basis for the set of all solutions to ($*$)
4. y_1 and y_2 are linearly dependent

116. Consider the partial differential equations

(i) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + (1 - \operatorname{sgn}(y))\frac{\partial^2 u}{\partial y^2} = 0$

(ii) $y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0$

Which of the following statements are true?

1. Equation (i) is parabolic for y > 0 and elliptic for y < 0
2. Equation (i) is hyperbolic for y > 0 and elliptic for y < 0
3. Equation (ii) is elliptic in I and III quadrant and hyperbolic in II and IV quadrant
4. Equation (ii) is hyperbolic in I and III quadrant and elliptic in II and IV quadrant

117. Consider the Cauchy problem

$$\begin{cases} \dfrac{\partial^2 u}{\partial x \partial y} = 0, \quad |x|<1, 0<y<1 \\ u(x,x^2)=0, \quad \dfrac{\partial u}{\partial y}(x,x^2)=g(x), |x|<1 \end{cases}$$

Which of the following statements are true?
1. A necessary condition for a solution to exist is that g is an odd function
2. A necessary condition for a solution to exist is that g is an even function
3. The solution (if it exists) is given by $u(x,y)=2\int_x^{\sqrt{y}} zg(z)dz$
4. The solution (if it exists) is given by $u(x,y)=2\int_{\sqrt{y}}^{x^2} zg(z)dz$

JUNE-2021

PART – B

118. Consider the following two initial value ODEs

(A) $\dfrac{dx}{dt}=x^3, x(0)=1;$

(B) $\dfrac{dx}{dt}=x\sin x^2, x(0)=2.$

Related to these ODEs, we make the following assertions.
I. The solution to (A) blows up in finite time
II. The solution to (B) blows up in finite time

Which of the following statements is true?
1. Both (I) and (II) are true
2. (I) is true but (II) is false
3. Both (I) and (II) are false
4. (I) is false but (II) is true

119. Let u(x, y) solve the Cauchy problem $\dfrac{\partial u}{\partial y} - x\dfrac{\partial u}{\partial x} + u - 1 = 0$ where $-\infty < x < \infty$, $y \ge 0$ and u(x, 0) = sinx. Then u(0, 1) is equal to

1. $1-\dfrac{1}{e}$
2. $1+\dfrac{1}{e}$
3. $1-\dfrac{1-\sin e}{e}$
4. $1+\dfrac{1-\sin e}{e}$

120. If y(x) is a solution of the equation 4xy″ + 2y′ + y = 0.
Satisfying y(0) = 1. Then y″(0) is equal to
1. 1/24
2. 1/12
3. 1/6
4. 1/2

121. Which of the following partial differential equations is NOT PARABOLIC for all x, $y \in \mathbb{R}$?

1. $x^2\dfrac{\partial^2 u}{\partial x \partial y} - 2xy\dfrac{\partial u}{\partial y} + y^2 = 0$
2. $x^2\dfrac{\partial^2 u}{\partial x^2} - 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 0$
3. $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 0$
4. $x^2\dfrac{\partial^2 u}{\partial x^2} - 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} + x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$

PART – C

122. Consider the Euler method for integration of the system of differential equations

$$\dot{x} = -y$$
$$\dot{y} = x$$

Assume that (x_i^n, y_i^n) are the points obtained for i = 0, 1, …, n^2 using a time-step h = 1/n starting at the initial point $(x_0, y_0) = (1, 0)$. Which of the following statements are true?
1. The points (x_i^n, y_i^n) lie on a circle of radius 1
2. $\lim_{n\to\infty}(x_n^n, y_n^n) = (\cos(1), \sin(1))$
3. $\lim_{n\to\infty}(x_2^n, y_2^n) = (1, 0)$
4. $(x_i^n)^2 + (y_i^n)^2 > 1$ for $i \ge 1$

123. Let u be a positive eigenfunction with eigenvalue λ for the boundary value problem

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$\ddot{u}+2\dot{u}+a(t)u=\lambda u,\ \dot{u}(0)=0=\dot{u}(1)$, where a : [0, 1] → (1, ∞) is a continuous function. Which of the following statements are possibly true?

1. $\lambda > 0$
2. $\lambda < 0$
3. $\int_0^1 (\dot{u})^2\,dt = 2\int_0^1 u\,\dot{u}\,dt + \int_0^1 (a(t)-\lambda)u^2\,dt$
4. $\lambda = 0$

124. Let u(x, y) solve the partial differential equation (PDE) $x^2\dfrac{\partial^2 u}{\partial x\partial y}+3y^2u=0$ with $u(x, 0) = e^{1/x}$.
Which of the following statements are true?

1. The PDE is not linear
2. $u(1, 1) = e^2$
3. $u(1, 1) = e^{-2}$
4. The method of separation of variables can be utilized to compute the solution u(x, y)

125. Which of the following expressions for u = u(x, t) are solutions of $u_t - e^{-t}u_x + u = 0$ with u(x, 0) = x?

1. $e^t(x + e^t - 1)$
2. $e^{-t}(x - e^{-t} + 1)$
3. $x - e^t + 1$
4. xe^t

126. Consider the 2^{nd} order ODE $\ddot{x}+p(t)\dot{x}+q(t)x=0$ and let x_1, x_2 be two solutions of this ODE in [a, b]. Which of the following statements are true for the Wronskian W of x_1, x_2?

1. W ≡ 0 in (a, b) implies that x_1, x_2 are independent
2. W can change sign in (a, b)
3. $W(t_0) = 0$ for some $t_0 \in (a, b)$ implies that W ≡ 0 in (a, b)
4. $W(t_0) = 1$ for some $t_0 \in (a, b)$ implies that W ≡ 1 in (a, b)

127. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be a non-zero smooth vector field satisfying div$f \neq 0$. Which of the following are necessarily true for ODE $\dot{x} = f(x)$?

1. There are no equilibrium points
2. There are no periodic solutions
3. All the solutions are bounded
4. All the solutions are unbounded

JUNE-2022

PART – B

128. Let $G : [0, 1] \times [0, 1] \to \mathbb{R}$ be defined as $G(t,x)=\begin{cases} t(1-x) & \text{if } t\le x\le 1\\ x(1-t) & \text{if } x\le t\le 1\end{cases}$. For a continuous function f on [0,1], define $I[f]=\int_0^1\int_0^1 G(t,x)\,f(t)\,f(x)\,dt\,dx$.
Which of the following is true?

1. I[f] > 0 if f is not identically zero
2. There exists non-zero f such that I[f] = 0
3. There is f such that I[f] < 0
4. $I[\sin(\pi x)] = 1$

129. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be continuous and
f(t, x) < 0 if tx > 0
f(t, x) > 0 if tx < 0
Consider the problem of solving the following
$\dot{x} = f(t, x)$, x(0) = 0
Which of the following is true?

1. There exists a unique local solution
2. There exists a local solution but may not be unique
3. There may not exist any solution
4. If local solution exists then it is unique.

130. Consider the second order PDE $au_{xx} + bu_{xy} + au_{yy} = 0$ in $\mathbb{R}^2$, for a, b ∈ ℝ. Which of the following is true?

1. The PDE is hyperbolic for b ≤ 2a
2. The PDE is parabolic for b ≤ 2a
3. The PDE is elliptic for |b| < 2|a|
4. The PDE is hyperbolic for |b| < 2|a|

131. Let u(x, t) be a smooth solution to the wave equation (*) $\dfrac{\partial^2 u}{\partial t^2}-\dfrac{\partial^2 u}{\partial x^2}=0$ for (x, t) $\in \mathbb{R}^2$. Which of the following is false?

1. u(x − θ, t) also solves the wave equation (*) for any fixed θ ∈ ℝ
2. $\dfrac{\partial u}{\partial x}$ also solves the wave equation (*)
3. u(3x, 9t) also solves the wave equation (*)
4. u(3x, 3t) also solves the wave equation (*)

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PART – C

132. Let u be a solution of following PDE
$u_x + xu_y = 0$
$u(x, 0) = e^x$
which statements are true

1. $u(2, 1) = e^2$
2. $u\left(1, \frac{1}{2}\right) = 1$
3. $u(-2, 1) = e^{-\sqrt{2}}$
4. $u(-2, 1) = e^{\sqrt{2}}$

133. Consider the two following initial value problem

$$(I)\quad y'(x) = y^{\frac{1}{3}},\ y(0) = 0 \qquad (II)\quad y'(x) = -y^{\frac{1}{3}},\ y(0) = 0$$

Which of the following statements are true
1. I is uniquely solvable
2. II is uniquely solvable
3. I has multiple solution
4. II has multiple solution

134. Consider the linear system y′ = Ay + h where $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$ and $h = \begin{pmatrix} 3t+1 \\ 2t+5 \end{pmatrix}$.
Suppose y(t) is a solution such that $\lim_{t\to\infty} \frac{y(t)}{t} = d \in \mathbb{R}^2$.
What is the value of d?

1. $\begin{pmatrix} -\frac{4}{3} \\ -\frac{5}{3} \end{pmatrix}$
2. $\begin{pmatrix} \frac{4}{3} \\ -\frac{5}{3} \end{pmatrix}$
3. $\begin{pmatrix} \frac{2}{3} \\ -\frac{5}{3} \end{pmatrix}$
4. $\begin{pmatrix} -\frac{2}{3} \\ -\frac{5}{3} \end{pmatrix}$

135. Let $A \in M_3(\mathbb{R})$ be skew-symmetric and let $x : [0, \infty) \to \mathbb{R}^3$ be a solution of $x'(t) = Ax(t)$, for all $t \in (0, \infty)$.
Which of the following statements are true?
1. $\|x(t)\| = \|x(0)\|$, for all $t \in (0, \infty)$.
2. For some $a \in \mathbb{R}^3\setminus\{0\}$, $\|x(t) - a\| = \|x(0) - a\|$, for all $t \in (0, \infty)$
3. $x(t) - x(0) \in \operatorname{im}A$, for all $t \in (0, \infty)$
4. $\lim_{t\to\infty} x(t)$ exists

JUNE-2023

PART – B

136. Let u(x, t) be the solution of
$u_{tt} - u_{xx} = 0$, $0 < x < 2, t > 0$,
$u(0, t) = 0 = u(2, t)$, $\forall\, t > 0$,
$u(x, 0) = \sin(\pi x) + 2\sin(2\pi x)$, $0 \le x \le 2$,
$u_t(x, 0) = 0$, $0 \le x \le 2$.
Which of the following is true?
(1) u(1, 1) = -1
(2) u(1/2, 1) = 0
(3) u(1/2, 2) = 1
(4) $u_t(1/2, 1/2) = \pi$

137. Let u(x, y) be the solution of the Cauchy problem
$uu_x + u_y = 0$, $x \in \mathbb{R}$, $y > 0$
$u(x, 0) = x$, $x \in \mathbb{R}$
Which of the following is the value of u(2, 3)?
(1) 2
(2) 3
(3) 1/2
(4) 1/3

138. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a locally Lipschitz function. Consider the initial value problem
$\dot{x} = f(t, x), x(t_0) = x_0$
for $(t_0, x_0) \in \mathbb{R}^2$. Suppose that $J(t_0, x_0)$ represents the maximal interval of existence for the initial value problem. Which of the following statements is true?
(1) $J(t_0, x_0) = \mathbb{R}$
(2) $J(t_0, x_0)$ is an open set
(3) $J(t_0, x_0)$ is a closed set
(4) $J(t_0, x_0)$ could be an empty set

139. Suppose x(t) is the solution of the following initial value problem in $\mathbb{R}^2$
$\dot{x} = Ax, x(0) = x_0$, where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
Which of the following statements is true?
(1) x(t) is a bounded solution for some $x_0 \neq 0$
(2) $e^{-6t}|x(t)| \to 0$ as $t \to \infty$, for all $x_0 \neq 0$
(3) $e^{-t}|x(t)| \to \infty$ as $t \to \infty$, for all $x_0 \neq 0$
(4) $e^{-10t}|x(t)| \to 0$ as $t \to \infty$, for all $x_0 \neq 0$

PART – C

140. Consider the following initial value problem (IVP),

$$\frac{du}{dt} = t^2 u^{\frac{1}{5}}, u(0) = 0.$$

Which of the following statements are correct?

(1) The function $g(t,u) = t^2 u^{\frac{1}{5}}$ does not satisfy the Lipschitz's condition with respect to u in any neighbourhood of u = 0
(2) There is no solution for the IVP
(3) There exist more than one solution for the IVP
(4) The function $g(t,u) = t^2 u^{\frac{1}{5}}$ satisfies the Lipschitz's condition with respect to u in some neighbourhood of u = 0 and hence there exists a unique solution for the IVP.

141. Let u: $\mathbb{R}^2 \to \mathbb{R}$ be the solution to the Cauchy problem:

$$\begin{cases} \partial_x u + 2\partial_y u = 0 & for\ (x,y) \in \mathbb{R}^2, \\ u(x,y) = \sin(x) & for\ y = 3x+1, x \in \mathbb{R} \end{cases}$$

Let v: $\mathbb{R}^2 \to \mathbb{R}$ be the solution to the Cauchy problem:

$$\begin{cases} \partial_x v + 2\partial_y v = 0 & for\ (x,y) \in \mathbb{R}^2, \\ v(x,0) = \sin(x) & for\ x \in \mathbb{R} \end{cases}$$

Let S = [0, 1] × [0, 1].
Which of the following statements are true?
(1) u changes sign in the interior of S.
(2) u(x, y) = v(x, y) along a line in S.
(3) v changes sign in the interior of S.
(4) v vanishes along a line in S.

142. Let us consider the follownig two initial value problems

$$(P) \begin{cases} x'(t) = \sqrt{x(t)}, & t > 0, \\ x(0) = 0, \end{cases} \quad \text{and}$$

$$(Q) \begin{cases} y'(t) = -\sqrt{y(t)}, & t > 0, \\ y(0) = 0. \end{cases}$$

Which of the following statements are true?
(1) (P) has a unique solution in [0, ∞).
(2) (Q) has a unique solution in [0, ∞).
(3) (P) has infinitely many solutions in [0, ∞).
(4) (Q) has infinitely many solutions in [0, ∞).

143. Let u = u(x, y) be the solution to the following Cauchy problem

$u_x + u_y = e^u$ for (x, y) $\in \mathbb{R} \times \left(0, \frac{1}{e}\right)$ and

u(x, 0) = 1 for x $\in \mathbb{R}$.
Which of the following statements are true?

(1) $u\left(\frac{1}{2e}, \frac{1}{2e}\right) = 1$

(2) $u_x\left(\frac{1}{2e}, \frac{1}{2e}\right) = 0$

(3) $u_y\left(\frac{1}{4e}, \frac{1}{4e}\right) = \log 4$

(4) $u_y\left(0, \frac{1}{4e}\right) = \frac{4e}{3}$

144. Let f $\in C^1(\mathbb{R})$ be bounded. Let us consider the initial value problem

$$(P) \begin{cases} x'(t) = f(x(t)), t > 0, \\ x(0) = 0. \end{cases}$$

Which of the following statements are true?
(1) (P) has solution (s) defined for all t > 0.
(2) (P) has a unique solution.
(3) (P) has infinitely many solutions.
(4) The solution(s) of (P) is/are Lipschitz.

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PART – B

145. Consider the Cauchy problem for the wave equation

$$\frac{\partial^2 u}{\partial t^2} - 4\frac{\partial^2 u}{\partial x^2} = 0, \quad -\infty < x < \infty, \quad t > 0,$$

$$u(x,0) = \begin{cases} e^{\left(-\frac{1}{x^2}\right)}, & x \neq 0, \\ 0, & x = 0, \end{cases}$$

$\frac{\partial u}{\partial t}(x,0) = xe^{-x^2}$, x $\in \mathbb{R}$.

Which one of the following is true?

(1) $\lim_{t\to\infty} u(5,t) = 1$

(2) $\lim_{t\to\infty} u(5,t) = 2$

(3) $\lim_{t\to\infty} u(5,t) = \frac{1}{2}$

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(4) $\lim_{t\to\infty} u(5,t) = 0$

146. The following partial differential equation

$$x^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x\partial y} - 3y^2\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$$

is

(1) elliptic in $\{(x, y) \in \mathbb{R}^2 : y > 0\}$
(2) parabolic in $\{(x, y) \in \mathbb{R}^2 : x > 0, y > 0\}$
(3) hyperbolic in $\{(x, y) \in \mathbb{R}^2 : xy \neq 0\}$
(4) parabolic in $\{(x, y) \in \mathbb{R}^2 : xy \neq 0\}$

147. The smallest real number λ for which the problem
$-y'' + 3y = \lambda y$, $y(0) = 0$, $y(\pi) = 0$
has a non-trivial solution is
(1) 3 (2) 2
(3) 1 (4) 4

PART – C

148. Consider the following initial value problem

$$y' = y + \frac{1}{2}|\sin(y^2)|, \qquad x > 0, \quad y(0) = -1$$

Which of the following statements are true?
(1) there exists an $\alpha \in (0, \infty)$ such that $\lim_{x\to\alpha^-} |y(x)| = \infty$
(2) $y(x)$ exists on $(0, \infty)$ and it is monotone
(3) $y(x)$ exists on $(0, \infty)$, but not bounded below
(4) $y(x)$ exists on $(0, \infty)$, but not bounded above

149. Let $B = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ be the open unit disc in $\mathbb{R}^2$, $\partial B = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ be its boundary and $\overline{B} = B \cup \partial B$. For $\lambda \in (0, \infty)$, let S_λ be the set of twice continuously differentiable functions in B, that are continuous on $\overline{B}$ and satisfy

$$\left(\frac{\partial u}{\partial x}\right)^2 + \lambda\left(\frac{\partial u}{\partial y}\right)^2 = 1, in\ B$$

$u(x, y) = 0$ on ∂B.
Then which of the following statements are true?
(1) $S_1 = \varnothing$
(2) $S_2 = \varnothing$
(3) S_1 has exactly one element and S_2 has exactly two elements
(4) S_1 and S_2 are both infinite

150. Consider the Cauchy problem

$$u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1, \quad (x, y) \in \mathbb{R} \times (0, \infty),$$

$u(x, 0) = kx$, $x \in \mathbb{R}$,
with a given real parameter k. For which of the following values of k does the above problem have a solution defined on $\mathbb{R} \times (0, \infty)$?
(1) k = 0 (2) k = -2
(3) k = 4 (4) k = 1

151. Consider the problem
$y' = (1 - y^2)^{10}\cos y$, $y(0) = 0$.
Let J be the maximal interval of existence and K be the range of the solution of the above problem. Then which of the following statements are true?
(1) $J = \mathbb{R}$ (2) $K = (-1, 1)$
(3) $J = (-1, 1)$ (4) $K = [-1, 1]$

152. Consider the initial value problem
$x^2y'' - 2x^2y' + (4x - 2)y = 0$, $y(0) = 0$.
Suppose $y = \varphi(x)$ is a polynomial solution satisfying $\varphi(1) = 1$. Which of the following statements are true?
(1) $\varphi(4) = 16$ (2) $\varphi(2) = 2$
(3) $\varphi(5) = 25$ (4) $\varphi(3) = 3$

ANSWERS

1. (3)	2. (2)	3. (2,3)
4. (1,3)	5. (2,3)	6. (1)
7. (2)	8. (2,4)	9. (2,3,4)
10. (2,4)	11. (3)	12. (3)
13. (3)	14. (1)	15. (2)
16. (2,3)	17. (2)	18. (3)
19. (1)	20. (1,4)	21. (1,4)
22. (1,2,4)	23. (4)	24. (2)
25. (3)	26. (1,2,3)	27. (3)
28. (1)	29. (4)	30. (1,2)
31. (1,2,3,4)	32. (1)	33. (3)
34. (1)	35. (4)	36. (2,4)
37. (1,2)	38. (2,3)	39. (3,4)
40. (3,4)	41. (1,3)	42. (1)
43. (1)	44. (1)	45. (2)
46. (2,4)	47. (1,2,4)	48. (1,3)
49. (1,3)	50. (1,3)	51. (1,3,4)
52. (2)	53. (1)	54. (1)
55. (3)	56. (2,3)	57. (1,3)
58. (3,4)	59. (1,3)	60. (3)
61. (3)	62. (3)	63. (1)
64. (1)	65. (3,4)	66. (1,2,3,4)
67. (1,3)	68. (1,2,4)	69. (2,3)
70. (2,4)	71. (3)	72. (4)
73. (4)	74. (3)	75. (3)
76. (1,2,3,4)	77. (1,2,3)	78. (1)
79. (1,3)	80. (3)	81. (3)
82. (3)	83. (3)	84. (2,4)
85. (1,2,3,4)	86. (1,4)	87. (1,3,4)
88. (2,3)	89. (1,2,3,4)	90. (4)
91. (2)	92. (2)	93. (3)
94. (1,4)	95. (1,3)	96. (1,2)
97. (2)	98. (1,3)	99. (2,3)
100. (3)	101. (4)	102. (4)
103. (4)	104. (1,2,3,4)	105. (1)
106. (2)	107. (2,3)	108. (1,2)
109. (3)	110. (4)	111. (3)
112. (1)	113. (2,3)	114. (1,3)
115. (2,3)	116. (2,4)	117. (2,4)
118. (2)	119. (1)	120. (2)
121. (1)	122. (2,3,4)	123. (2,3,4)
124. (2,4)	125. (2)	126. (3)
127. (1,2,4)	128.	129. (1)
130. (3)	131. (3)	132. (4)
133.	134.	135.
136. (2)	137. (2)	138. (2)
139. (4)	140. (1,3)	141. (2,3,4)
142. (2,3)	143. (2,4)	144. (1,2,4)
145. (1)	146. (3)	147. (4)
148.	149.	150. (1,3,4)
151. (1,2)	152. (1,3)	

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