INTEGRAL EQUATION ASSIGNMENT

DEC - 2014

1. Let $y:[0,\infty)\to\mathbb{R}$ be twice continuously differentiable and satisfy
$y(x)+\int_0^x (x-s)y(s)ds = x^3/6$. Then

1. $y(x)=\frac{1}{6}\int_0^x s^3\sin(x-s)ds$
2. $y(x)=\frac{1}{6}\int_0^x s^3\cos(x-s)ds$
3. $y(x)=\int_0^x s\sin(x-s)ds$
4. $y(x)=\int_0^x s\cos(x-s)ds$

2. Let $u\in C^2([0,1])$ satisfy for some $\lambda\neq 0$ and $a\neq 0$
$u(x)+\frac{\lambda}{2}\int_0^1 |x-s|\,u(s)ds = ax+b.$
Then u also satisfies

1. $\frac{d^2u}{dx^2}+\lambda u=0$
2. $\frac{d^2u}{dx^2}-\lambda u=0$
3. $\frac{du}{dx}-\frac{\lambda}{2}\int_0^1 \frac{x-s}{|x-s|}u(s)ds=a$
4. $\frac{du}{dx}+\frac{\lambda}{2}\int_0^1 \frac{x-s}{|x-s|}u(s)ds=a$

JUNE-2015

3. The integral equation
$y(x)=\lambda\int_0^1 (3x-2)ty(t)dt$, with λ as a parameter, has

1. only one characteristic number
2. two characteristic numbers
3. more than two characteristic numbers
4. no characteristic number

4. For the integral equation
$y(x)=1+x^3+\int_0^x K(x,t)y(t)dt$ with kernel $K(x,t) = 2^{x-t}$, the iterated kernel $K_3(x,t)$ is

1. $2^{x-t}(x - t)^2$
2. $2^{x-t}(x - t)^3$
3. $2^{x-t-1}(x - t)^2$
4. $2^{x-t-1}(x - t)^3$

DEC-2015

5. The resolvent kernel R (x, t, λ) for the Volterra integral equation $\varphi(x)=x+\lambda\int_a^x \varphi(s)\,ds$, is

1. $e^{\lambda(x+t)}$
2. $e^{\lambda(x-t)}$
3. $\lambda e^{(x+t)}$
4. $e^{\lambda xt}$

6. Let y : [0, ∞) → [0, ∞) be a continuously differentiable function satisfying
$y(t)=y(0)+\int_0^t y(s)\,ds$ for t ≥ 0. Then

1. $y^2(t)=y^2(0)+\int_0^t y^2(s)\,ds.$
2. $y^2(t)=y^2(0)+2\int_0^t y^2(s)\,ds.$
3. $y^2(t)=y^2(0)+\int_0^t y\,(s)\,ds.$
4. $y^2(t)=y^2(0)+\left(\int_0^t y(s)\,ds\right)^2+2y(0)\int_0^t y(s)\,ds.$

7. Let λ_1, λ_2 be the characteristic numbers and f_1, f_2 be the corresponding eigen functions for the homogenous integral equation
$\varphi(x)-\lambda\int_0^1 (2xt+4x^2)\varphi(t)\,dt=0.$ Then

1. $\lambda_1 \neq \lambda_2$
2. $\lambda_1 = \lambda_2$
3. $\int_0^1 f_1(x)f_2(x)\,dx=0$
4. $\int_0^1 f_1(x)f_2(x)\,dx=1$

JUNE – 2016

PART – B

8. Consider the integral equation
$y(x)=x^3+\int_0^x Sin(x-t)y(t)dt, x\in[0,\pi].$
Then the value of y(1) is

1. 19/20
2. 1
3. 17/20
4. 21/20

PART- C

9. The curve y = y(x), passing through the point $(\sqrt{3},1)$ and defined by the following property (Volterra integral equation of the first kind) $\int_0^y \frac{f(v)dv}{\sqrt{y-v}}=4\sqrt{y}$, where $f(y)=\sqrt{1+\frac{1}{y'^2}}$, is the part of a

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1.Straight line. 2. Circle
3. Parabola. 4. Cycloid.

DEC-2016

10. Let ϕ satisfy $\phi(x) = f(x) + \int_0^x \sin(x-t)\phi(t)\,dt$.

Then ϕ is given by

1. $\phi(x) = f(x) + \int_0^x (x-t) f(t)\,dt$
2. $\phi(x) = f(x) - \int_0^x (x-t) f(t)\,dt$
3. $\phi(x) = f(x) - \int_0^x \cos(x-t) f(t)\,dt$
4. $\phi(x) = f(x) - \int_0^x \sin(x-t) f(t)\,dt$

11. Which of the following are the characteristic numbers and the corresponding eigen-functions for the Fredholm homogeneous equation whose kernel is

$$K(x,t) = \begin{cases} (x+1)t, & 0 \le x \le t \\ (t+1)x, & t \le x \le 1 \end{cases} ?$$

1. 1, e^x
2. $-\pi^2$, $\pi \sin \pi x + \cos \pi x$
3. $-4\pi^2$, $\pi \sin \pi x + \pi \cos 2\pi x$
4. $-\pi^2$, $\pi \cos \pi x + \sin \pi x$

12. The integral equation

$$\phi(x) - \frac{2}{\pi}\int_0^{\pi} \cos(x+t)\phi(t)dt = f(x)$$

has infinitely many solutions if

1. f(x) = cosx 2. f(x) = cos 3x
3. f(x) = sin x 4. f(x) = sin 3x

JUNE – 2017

13. Let $\phi(x)$ be the solution of

$\int_0^x e^{x-t}\phi(t)\,dt = x, \quad x > 0$. Then $\phi(1)$ equals

1. -1 2. 0 3. 1 4. 2

14. Let y(x) be the solution of the integral equation $y(x) = x - \int_0^x xt^2 y(t)dt, x > 0.$

Then the value of the function y(x) at $x = \sqrt{2}$ is equal to

1. $\frac{1}{\sqrt{2e}}$ 2. $\frac{e}{2}$
3. $\frac{\sqrt{2}}{e^2}$ 4. $\frac{\sqrt{2}}{e}$

15. The solutions for λ = -1 and λ = 3 of the integral equation

$y(x) = 1 + \lambda\int_0^1 K(x,t)y(t)dt,$ where

$K(x,t) = \begin{cases} \cosh x \sinh t, & 0 \le x \le t \\ \cosh t \sinh t, & t \le x \le 1 \end{cases}$ are, respectively,

1. $-\frac{x^2}{2} + \frac{3}{2} - \tanh 1$ and $\frac{1}{4}\left(\frac{3\cos 2x}{\cos 2 - 2\sin 2\tanh 1} + 1\right)$
2. $-\frac{x^2}{2} + \frac{3}{2} - \tanh 1$ and $\frac{1}{4}\left(\frac{3\cos 2x}{\cosh 2 - 2\sinh 2\tanh 1} + 1\right)$
3. $\frac{x^2}{2} + \frac{3}{2} - \tanh 1$ and $\frac{1}{4}\left(\frac{3\cos 2x}{\cosh 2 - 2\sinh 2\tanh 1} - 1\right)$
4. $\frac{x^2}{2} + \frac{3}{2} - \tanh 1$ and $\frac{1}{4}\left(\frac{3\cos 2x}{\cos 2 - 2\sin 2\tanh 1} - 1\right)$

JUNE – 2018

PART - B

16. The resolvent kernel for the integral equation $\phi(x) = x^2 + \int_0^x e^{t-x}\varphi(t)dt$ is

1. e^{t-x} 2. 1
3. e^{x-t} 4. $x^2 + e^{x-t}$

PART - C

17. The values of λ for which the following equation has a non-trivial solution

$\phi(x) = \lambda\int_0^{\pi} K(x,t)\phi(t)\,dt, 0 \le x \le \pi,$

where

$K(x,t) = \begin{cases} \sin x \cos t, & 0 \le x \le t \\ \cos x \sin t, & t \le x \le \pi \end{cases}$ are

1. $\left(n + \frac{1}{2}\right)^2 - 1$, $n \in \mathbb{N}$

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2. $n^2 - 1, n \in \mathbb{N}$
3. $\frac{1}{2}(n+1)^2 - 1, n \in \mathbb{N}$
4. $\frac{1}{2}(2n+1)^2 - 1, n \in \mathbb{N}$

18. Consider the integral equation

$$\phi(x) = \lambda \int_0^{\pi} [\cos x \cos t - 2\sin x \sin t]\phi(t)\,dt + \cos 7x,\ 0 \le x \le \pi$$

Which of the following statements are true?

1. For every $\lambda \in \mathbb{R}$, a solution exists
2. There exists $\lambda \in \mathbb{R}$ such that solution does not exist
3. There exists $\lambda \in \mathbb{R}$ such that there are more than one but finitely many solutions
4. There exists $\lambda \in \mathbb{R}$ such that there are infinitely many solutions

DECEMBER – 2018

PART - B

19. If φ is the solution of

$$\int_0^x (1 - x^2 + t^2)\varphi(t)\,dt = \frac{x^2}{2},$$ then $\varphi(\sqrt{2})$ is equal to

1. $\sqrt{2}e^{\sqrt{2}}$
2. $\sqrt{2}e^2$
3. $\sqrt{2}e^{2\sqrt{2}}$
4. $2e^4$

PART-C

20. If φ is the solution of $\varphi(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x-t) - 4(x-t)^2]\varphi(t)\,dt$, then $\varphi(\log 2)$ is equal to

1. 2
2. 4
3. 6
4. 8

21. A characteristic number and the corresponding eigenfunction of the homogenous Fredholm integral equation with kernel $K(x,t) = \begin{cases} x(t-1), & 0 \le x \le t \\ t(x-1), & t \le x \le 1 \end{cases}$ are

1. λ - $-\pi^2$, $\varphi(x) = \sin\pi x$
2. $\lambda = -2\pi^2$, $\varphi(x) = \sin 2\pi x$
3. $\lambda = -3\pi^2$, $\varphi(x) = \sin 3\pi x$
4. $\lambda = -4\pi^2$, $\varphi(x) = \sin 2\pi x$

JUNE – 19

PART – B

22. If y is a solution of $y(x) - \int_0^x (x-t)\,y(t)\,dt = 1,$ then which of the following is true?

1. y is bounded but not periodic in $\mathbb{R}$
2. y is periodic in $\mathbb{R}$
3. $\int_{\mathbb{R}} y(x)\,dx < \infty$
4. $\int_{\mathbb{R}} \frac{dx}{y(x)} < \infty$

PART – C

23. Consider the integral equation $\varphi(x) - \frac{e}{2}\int_{-1}^{1} xe^t \varphi(t)\,dt = f(x).$ Then

1. there exists a continuous function $f : [-1, 1] \to (0, \infty)$ for which solution exists
2. there exists a continuous function $f : [-1, 1] \to (-\infty, 0)$ for which solution exists
3. for $f(x) = e^{-x}(1 - 3x^2)$, a solution exists
4. for $f(x) = e^{-x}(x + x^3 + x^5)$, a solution exists

DECEMBER – 2019

PART - B

24. Let ϕ be the solution of

$$\phi(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x-t) - 4(x-t)^2]\phi(t)\,dt.$$

Then $\phi(1)$ is equal to

1. e^{-1}
2. e^{-2}
3. e
4. e^2

PART – C

25. Assume that h_1, h_2, g_1 and $g_2 \in C([a, b])$. Let

$$\phi(x) = f(x) + \lambda \int_a^b [h_1(t)g_1(x) + h_2(t)g_2(x)]\phi(t)\,dt$$

be an integral domain. Consider the following statements:

S_1 : If the given interval equation has a solution for some $f \in C([a, b])$, then

$$\int_a^b f(t)\,g_1(t)\,dt = 0 = \int_a^b f(t)\,g_2(t)dt.$$

S_2 : The given integral equation has a unique solution for every f ∈ C([a, b]) if λ is not a characteristic number of the corresponding homogeneous equation.

Then
1. Both S_1 and S_2 are true
2. S_1 is true but S_2 is false
3. S_1 is false but S_2 is true
4. Both S_1 and S_2 are false

26. The integral equation

$$\phi(x) = 1 + \frac{2}{\pi}\int_0^{\pi} (\cos^2 x)\phi(t)\,dt$$

has
1. no solution
2. unique solution
3. more than one but finitely many solutions
4. infinitely many solutions

JUNE – 20

PART – B

27. The solution of the Fredholm integral equation

$y(s) = s + 2\int_0^1 (st^2 + s^2 t)\,y(t)\,dt$ is

1. y(s) = -(50s + 40s^2)
2. y(s) = (30s + 15s^2)
3. y(s) = -(30s + 40s^2)
4. y(s) = (60s + 50s^2)

PART – C

28. For the Fredholm integral equation

$$y(s) = \lambda\int_0^1 e^s e^t y(t)dt$$

Which of the following statements are true?
1. It has a non-trivial solution satisfying $\int_0^1 e^t y(t)dt = 0$
2. Only the trivial solution satisfies $\int_0^1 e^t y(t)dt = 0$
3. It has non-trivial solution for all λ ≠ 0
4. It has non-trivial solutions only if $\lambda = \frac{2}{e^2 - 1}$ and $\int_0^1 e^t y(t)\,dt \neq 0$

JUNE – 21

PART – B

29. Consider the integral equation $\int_0^x (x-t)u(t)\,dt = x; x \geq 0$ for continuous functions u defined on [0, ∞). The equation has
1. A unique bounded solution
2. No solution
3. More than one solution u such that |u(x)| ≤ C(1 + |x|) for some constant C
4. A unique solution u such that |u(x)| ≤ C(1 + |x|) for some constant C

PART – C

30. Let K(x, y) be a kernel in [0, 1] × [0, 1], defined as K(x, y) = sin(2πx) sin(2πy). Consider the integral operator

$K(u)(x) = \int_0^1 u(y)\,K(x,y)\,dy$ where u ∈ C ([0, 1]). Which of the following assertions on K are true?
1. The null space of K is infinite dimensional
2. $\int_0^1 v(x)\,K(u)(x)\,dx = \int_0^1 K(v)(x)\,u(x)\,dx$ for all u, v ∈ C([0, 1])
3. K has no negative eigenvalue
4. K has an eigenvalue greater than 3/4

JUNE – 22

PART – B

31. For any two continuous functions f, g : ℝ → ℝ define

$f * g(t)\int_0^t f(s)\,g(t-s)\,ds.$ Which of the the following is the value of f ∗ g(t) when f(t) = exp (-t) and g(t) = sin (t).

1. $\frac{1}{2}[\exp(-t) + \sin(t) - \cos(t)]$
2. $\frac{1}{2}[-\exp(-t) + \sin(t) - \cos(t)]$
3. $\frac{1}{2}[\exp(-t) - \sin(t) - \cos(t)]$
4. $\frac{1}{2}[\exp(-t) + \sin(t) + \cos)t)]$

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PART – C

32. Let g be the solution of the Volterra type integral equation

$$g(s) = 1 + \int_0^s (s-t)\, g(t)\, dt; \ \ for\ \ all\ \ s \geq 0.$$

What are the possible values of g(1)?

1. 2e
2. $e - \frac{1}{e}$
3. $e + \frac{1}{e}$
4. $\frac{2}{e}$

33. Consider the following system of Integral

$$\varphi_1(x) = \sin x + \int_0^x \varphi_2(t)\, dt$$

$$\varphi_2(x) = 1 - \cos x - \int_0^x \varphi_1(t)\, dt$$

Which of the following statements are true

1. φ_1 vanishes atmost countably many points
2. φ_1 vanishes at uncountably many points
3. φ_2 vanishes at atmost countably many points
4. φ_2 vanishes at uncountably many points

JUNE – 23

PART – B

34. For the unknown y: [0, 1] → ℝ, consider the following two-point boundary value problem:

$$\begin{cases} y''(x) + 2y(x) = 0 & for\ x \in (0,1), \\ y(0) = y(1) = 0. & \end{cases}$$

It is given that the above boundary value problem corresponds to the following integral equation:

$$y(x) = 2\int_0^1 K(x,t)\, y(t)\, dt \qquad for\ \ x \in [0,1].$$

Which of the following is the kernel K(x, t)?

1. $K(x,t) = \begin{cases} t(1-x) & for\ t < x \\ x(1-t) & for\ t > x \end{cases}$
2. $K(x,t) = \begin{cases} t^2(1-x) & for\ t < x \\ x^2(1-t) & for\ t > x \end{cases}$
3. $K(x,t) = \begin{cases} \sqrt{t}(1-x) & for\ t < x \\ \sqrt{x}(1-t) & for\ t > x \end{cases}$
4. $K(x,t) = \begin{cases} \sqrt{t^3}(1-x) & for\ t < x \\ \sqrt{x^3}(1-t) & for\ t > x \end{cases}$

PART – C

35. Let $\lambda_1 < \lambda_2$ be two real characteristic numbers for the following homogeneous integral equation:

$$\varphi(x) = \lambda \int_0^{2\pi} \sin(x+t)\, \varphi(t)\, dt;$$

and let $\mu_1 < \mu_2$ be two real characteristic numbers for the following homogeneous integral equation:

$$\psi(x) = \mu \int_0^{\pi} \cos(x+t)\, \psi(t)\, dt.$$

Which of the following statements are true?

1. $\mu_1 < \lambda_1 < \lambda_2 < \mu_2$
2. $\lambda_1 < \mu_1 < \mu_2 < \lambda_2$
3. $|\mu_1 - \lambda_1| = |\mu_2 - \lambda_2|$
4. $|\mu_1 - \lambda_1| = 2|\mu_2 - \lambda_2|$

DECEMBER – 23

PART – B

36. The value of λ for which the integral equation

$$y(x) = \lambda \int_0^1 x^2 e^{x+t}\, y(t)\, dt$$

has a non-zero solution, is

(1) $\frac{4}{1+e^2}$

(2) $\frac{2}{1+e^2}$

(3) $\frac{4}{e^2-1}$

(4) $\frac{2}{e^2-1}$

PART – C

37. Consider the following Fredholm integral equation

$$y(x) - 3\int_0^1 tx\, y(t)\, dt = f(x),$$

where f(x) is a continuous function defined on the interval [0, 1]. Which of the following choices for f(x) have the property that the above integral equation admits at least one solution?

(1) $f(x) = x^2 - \frac{1}{2}$

(2) $f(x) = e^x$

(3) $f(x) = 2 - 3x$

(4) $f(x) = x - 1$

38. Let y be the solution to the Volterra integral equation

$$y(x) = e^x + \int_0^x \frac{1+x^2}{1+t^2}\, y(t)\, dt.$$

Then which of the following statements are true?

(1) $y(1) = \left(1 + \frac{\pi}{4}\right)e$

(2) $y(1) = \left(1 + \frac{\pi}{2}\right)e$

(3) $y(\sqrt{3}) = \left(1 + \frac{3\pi}{4}\right)e^{\sqrt{3}}$

(4) $y(\sqrt{3}) = \left(1 + \frac{4\pi}{3}\right)e^{\sqrt{3}}$

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ANSWERS

1. (3)	2. (1,4)	3. (4)
4. (3)	5. (2)	6. (2,4)
7. (1,3)	8. (4)	9. (1)
10. (1)	11. (1,4)	12. (2,3,4)
13. (2)	14. (4)	15. (2)
16. (2)	17. (1)	18. (1,4)
19. (2)	20. (1)	21. (1,4)
22. (4)	23. (3,4)	24. (3)
25. (3)	26. (1)	27. (3)
28. (2,4)	29. (2)	30. (1,2,3)
31. (3)	32.	33.
34. (1)	35. (1,3)	36. (3)
37. (1,3)	38. (2,4)	

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