GATE 2022 Mathematics (MA)

Useful data

$A\backslash B$	$\{a \in A \ : \ a \notin B\}$
$\mathbb{C}$	Set of all complex numbers
$\mathbb{C}^{m\times n}$	Set of all matrices of order $m \times n$ with complex entries
$\mathbb{C}^{\infty}(\Omega)$	Collection of all infinitely differentiable functions on the open domain Ω
i	$\sqrt{-1}$
I	Identity matrix of appropriate order
$L^2(\mathbb{R})$	$:= L^2(\mathbb{R}, dx)$
$L^2[a,b]$	$:= L^2([a,b], dx)$
$\mathbb{N}$	Set of all positive integers
$\mathbb{Q}$	Set of all rational numbers
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^{m\times n}$	Set of all matrices of order $m \times n$ with real entries
$\mathbb{S}^1$	$\{(x_1, x_2) \in \mathbb{R}^2 \ : \ x_1^2 + x_2^2 = 1\}$
$\mathbb{S}^2$	$\{(x_1, x_2, x_3) \in \mathbb{R}^3 \ : \ x_1^2 + x_2^2 + x_3^2 = 1\}$
$\mathbb{Z}$	Set of all integers

GATE 2022 General Aptitude (GA)

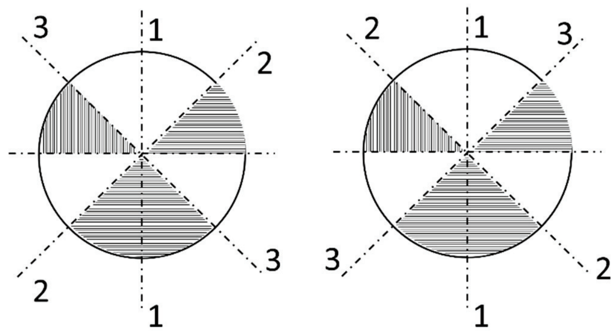
Q.1 – Q.5 Carry ONE mark each.

Q.1	As you grow older, an injury to your _________ may take longer to _________.
(A)	heel / heel
(B)	heal / heel
(C)	heal / heal
(D)	heel / heal

Q.2	In a 500 m race, P and Q have speeds in the ratio of $3 : 4$. Q starts the race when P has already covered 140 m. What is the distance between P and Q (in m) when P wins the race?
(A)	20
(B)	40
(C)	60
(D)	140

Q.3	Three bells P, Q, and R are rung periodically in a school. P is rung every 20 minutes; Q is rung every 30 minutes and R is rung every 50 minutes. If all the three bells are rung at 12:00 PM, when will the three bells ring together again the next time?
(A)	5:00 PM
(B)	5:30 PM
(C)	6:00 PM
(D)	6:30 PM

Q.4	Given below are two statements and four conclusions drawn based on the statements. Statement 1: Some bottles are cups. Statement 2: All cups are knives. Conclusion I: Some bottles are knives. Conclusion II: Some knives are cups. Conclusion III: All cups are bottles. Conclusion IV: All knives are cups. Which one of the following options can be logically inferred?
(A)	Only conclusion I and conclusion II are correct
(B)	Only conclusion II and conclusion III are correct
(C)	Only conclusion II and conclusion IV are correct
(D)	Only conclusion III and conclusion IV are correct

Q.5	The figure below shows the front and rear view of a disc, which is shaded with identical patterns. The disc is flipped once with respect to any one of the fixed axes 1-1, 2-2 or 3-3 chosen uniformly at random. What is the probability that the disc **DOES NOT** retain the same front and rear views after the flipping operation?  Front View Rear View
(A)	0
(B)	$\frac{1}{3}$
(C)	$\frac{2}{3}$
(D)	1

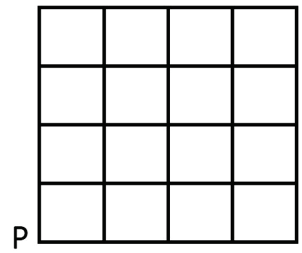
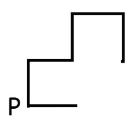
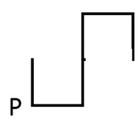
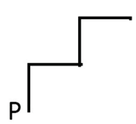
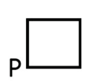
Q. 6 – Q. 10 Carry TWO marks each.

Q.6	Altruism is the human concern for the wellbeing of others. Altruism has been shown to be motivated more by social bonding, familiarity and identification of belongingness to a group. The notion that altruism may be attributed to empathy or guilt has now been rejected. Which one of the following is the CORRECT logical inference based on the information in the above passage?
(A)	Humans engage in altruism due to guilt but not empathy
(B)	Humans engage in altruism due to empathy but not guilt
(C)	Humans engage in altruism due to group identification but not empathy
(D)	Humans engage in altruism due to empathy but not familiarity

Q.7	There are two identical dice with a single letter on each of the faces. The following six letters: Q, R, S, T, U, and V, one on each of the faces. Any of the six outcomes are equally likely. The two dice are thrown once independently at random. What is the probability that the outcomes on the dice were composed only of any combination of the following possible outcomes: Q, U and V?
(A)	$\frac{1}{4}$
(B)	$\frac{3}{4}$
(C)	$\frac{1}{6}$
(D)	$\frac{5}{36}$

Q.8	The price of an item is 10% cheaper in an online store S compared to the price at another online store M. Store S charges ₹ 150 for delivery. There are no delivery charges for orders from the store M. A person bought the item from the store S and saved ₹ 100. What is the price of the item at the online store S (in ₹) if there are no other charges than what is described above?
(A)	2500
(B)	2250
(C)	1750
(D)	1500

Q.9	The letters P, Q, R, S, T and U are to be placed one per vertex on a regular convex hexagon, but not necessarily in the same order. Consider the following statements: • The line segment joining R and S is longer than the line segment joining P and Q. • The line segment joining R and S is perpendicular to the line segment joining P and Q. • The line segment joining R and U is parallel to the line segment joining T and Q. Based on the above statements, which one of the following options is CORRECT?
(A)	The line segment joining R and T is parallel to the line segment joining Q and S
(B)	The line segment joining T and Q is parallel to the line joining P and U
(C)	The line segment joining R and P is perpendicular to the line segment joining U and Q
(D)	The line segment joining Q and S is perpendicular to the line segment joining R and P

Q.10	 An ant is at the bottom-left corner of a grid (point P) as shown above. It aims to move to the top-right corner of the grid. The ant moves only along the lines marked in the grid such that the current distance to the top-right corner strictly decreases. Which one of the following is a part of a possible trajectory of the ant during the movement?
(A)	
(B)	
(C)	
(D)	

Q.11 – Q.35 Carry ONE mark each.

Q.11	Suppose that the characteristic equation of $M \in \mathbb{C}^{3\times 3}$ is $$\lambda^3 + \alpha\lambda^2 + \beta\lambda - 1 = 0,$$ where $\alpha, \beta \in \mathbb{C}$ with $\alpha + \beta \neq 0$. Which of the following statements is TRUE?
(A)	$M(I - \beta M) = M^{-1}(M + \alpha I)$
(B)	$M(I + \beta M) = M^{-1}(M - \alpha I)$
(C)	$M^{-1}(M^{-1} + \beta I) = M - \alpha I$
(D)	$M^{-1}(M^{-1} - \beta I) = M + \alpha I$

Q.12	Consider **P**: Let $M \in \mathbb{R}^{m \times n}$ with $m > n \geq 2$. If $\text{rank}(M) = n$, then the system of linear equations $Mx = 0$ has $x = 0$ as the only solution. **Q**: Let $E \in \mathbb{R}^{n \times n}, n \geq 2$ be a non-zero matrix such that $E^3 = 0$. Then $I + E^2$ is a singular matrix. Which of the following statements is TRUE?
(A)	Both **P** and **Q** are TRUE
(B)	Both **P** and **Q** are FALSE
(C)	**P** is TRUE and **Q** is FALSE
(D)	**P** is FALSE and **Q** is TRUE

Q.13	Consider the real function of two real variables given by $$u(x,y) = e^{2x}[\sin 3x \cos 2y \cosh 3y - \cos 3x \sin 2y \sinh 3y].$$ Let $v(x,y)$ be the harmonic conjugate of $u(x,y)$ such that $v(0,0) = 2$. Let $z = x+iy$ and $f(z) = u(x,y) + iv(x,y)$, then the value of $4 + 2if(i\pi)$ is
(A)	$e^{3\pi} + e^{-3\pi}$
(B)	$e^{3\pi} - e^{-3\pi}$
(C)	$-e^{3\pi} + e^{-3\pi}$
(D)	$-e^{3\pi} - e^{-3\pi}$

Q.14	The value of the integral $$\int_C \frac{z^{100}}{z^{101}+1}dz$$ where C is the circle of radius 2 centred at the origin taken in the anti-clockwise direction is
(A)	$-2\pi i$
(B)	2π
(C)	0
(D)	$2\pi i$

Q.15	Let X be a real normed linear space. Let $X_0 = \{x \in X : \|x\| = 1\}$. If X_0 contains two distinct points x and y and the line segment joining them, then, which of the following statements is TRUE?
(A)	$\|x+y\| = \|x\| + \|y\|$ and x, y are linearly independent
(B)	$\|x+y\| = \|x\| + \|y\|$ and x, y are linearly dependent
(C)	$\|x+y\|^2 = \|x\|^2 + \|y\|^2$ and x, y are linearly independent
(D)	$\|x+y\| = 2\|x\|\|y\|$ and x, y are linearly dependent

Q.16	Let $\{e_k : k \in \mathbb{N}\}$ be an orthonormal basis for a Hilbert space H. Define $f_k = e_k + e_{k+1}, k \in \mathbb{N}$ and $g_j = \sum_{n=1}^{j} (-1)^{n+1} e_n, j \in \mathbb{N}$. Then $\sum_{k=1}^{\infty} \lvert\langle g_j, f_k\rangle\rvert^2 =$
(A)	0
(B)	j^2
(C)	$4j^2$
(D)	1

Q.17	Consider $\mathbb{R}^2$ with the usual metric. Let $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and $B = \{(x, y) \in \mathbb{R}^2 : (x-2)^2+y^2 \leq 1\}$. Let $M = A \cup B$ and $N = \text{interior}(A) \cup \text{interior}(B)$. Then, which of the following statements is TRUE?
(A)	M and N are connected
(B)	Neither M nor N is connected
(C)	M is connected and N is not connected
(D)	M is not connected and N is connected

Q.18	The real sequence generated by the iterative scheme $$x_n = \frac{x_{n-1}}{2} + \frac{1}{x_{n-1}}, \quad n \geq 1$$
(A)	converges to $\sqrt{2}$, for all $x_0 \in \mathbb{R} \setminus \{0\}$
(B)	converges to $\sqrt{2}$, whenever $x_0 > \sqrt{\frac{2}{3}}$
(C)	converges to $\sqrt{2}$, whenever $x_0 \in (-1, 1) \setminus \{0\}$
(D)	diverges for any $x_0 \neq 0$

Q.19	The initial value problem $$\frac{dy}{dx} = \cos(xy),\ x \in \mathbb{R},\ y(0) = y_0,$$ where y_0 is a real constant, has
(A)	a unique solution
(B)	exactly two solutions
(C)	infinitely many solutions
(D)	no solution

Q.20	If eigenfunctions corresponding to distinct eigenvalues λ of the Sturm-Liouville problem $$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = \lambda y,\ 0 < x < \pi,$$ $$y(0) = y(\pi) = 0$$ are orthogonal with respect to the weight function $w(x)$, then $w(x)$ is
(A)	e^{-3x}
(B)	e^{-2x}
(C)	e^{2x}
(D)	e^{3x}

Q.21	The steady state solution for the heat equation $$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0,\ 0 < x < 2,\ t > 0,$$ with the initial condition $u(x,0) = 0,\ 0 < x < 2$ and the boundary conditions $u(0,t) = 1$ and $u(2,t) = 3,\ t > 0$, at $x = 1$ is
(A)	1
(B)	2
(C)	3
(D)	4

Q.22	Consider $([0,1], T_1)$, where T_1 is the subspace topology induced by the Euclidean topology on $\mathbb{R}$, and let T_2 be *any* topology on $[0,1]$. Consider the following statements: **P :** If T_1 is a proper subset of T_2, then $([0,1], T_2)$ is not compact. **Q :** If T_2 is a proper subset of T_1, then $([0,1], T_2)$ is not Hausdorff. Then
(A)	**P** is TRUE and **Q** is FALSE
(B)	Both **P** and **Q** are TRUE
(C)	Both **P** and **Q** are FALSE
(D)	**P** is FALSE and **Q** is TRUE

Q.23	Let $p : ([0,1], T_1) \to (\{0,1\}, T_2)$ be the quotient map, arising from the characteristic function on $[\frac{1}{2}, 1]$, where T_1 is the subspace topology induced by the Euclidean topology on $\mathbb{R}$. Which of the following statements is TRUE?
(A)	p is an open map but not a closed map
(B)	p is a closed map but not an open map
(C)	p is a closed map as well as an open map
(D)	p is neither an open map nor a closed map

Q.24	Set $X_n := \mathbb{R}$ for each $n \in \mathbb{N}$. Define $Y := \prod_{n\in\mathbb{N}} X_n$. Endow Y with the product topology, where the topology on each X_n is the Euclidean topology. Consider the set $$\Delta = \{(x, x, x, \cdots) \mid x \in \mathbb{R}\}$$ with the subspace topology induced from Y. Which of the following statements is TRUE?
(A)	Δ is open in Y
(B)	Δ is locally compact
(C)	Δ is dense in Y
(D)	Δ is disconnected

Q.25	Consider the linear sytem of equations $Ax = b$ with $$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 4 & 1 \\ 2 & 0 & 3 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}.$$ Which of the following statements are TRUE?
(A)	The Jacobi iterative matrix is $\begin{pmatrix} 0 & 1/4 & 1/3 \\ 1/3 & 0 & 1/3 \\ 2/3 & 0 & 0 \end{pmatrix}$
(B)	The Jacobi iterative method converges for any initial vector
(C)	The Gauss-Seidel iterative method converges for any initial vector
(D)	The spectral radius of the Jacobi iterative matrix is less than 1

Q.26	The number of non-isomorphic abelian groups of order $2^2.3^3.5^4$ is ________.

Q.27	The number of subgroups of a cyclic group of order 12 is __________.

Q.28	The radius of convergence of the series $$\sum_{n\geq 0} 3^{n+1} z^{2n}, \ z \in \mathbb{C}$$ is __________ (round off to TWO decimal places).

Q.29	The number of zeros of the polynomial $$2z^7 - 7z^5 + 2z^3 - z + 1$$ in the unit disc $\{z \in \mathbb{C} : \lvert z \rvert < 1\}$ is ________.

Q.30	If $P(x)$ is a polynomial of degree 5 and $$\alpha = \sum_{i=0}^{6} P(x_i) \left(\prod_{j=0,\, j \neq i}^{6} (x_i - x_j)^{-1} \right),$$ where $x_0,\ x_1,\ \cdots,\ x_6$ are distinct points in the interval $[2, 3]$, then the value of $\alpha^2 - \alpha + 1$ is ________.

Q.31	The maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$ is ______.

Q.32	If the function $f(x,y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}, x \neq 0, y \neq 0$ attains its local minimum value at the point (a, b), then the value of $a^3 + b^3$ is __________ (round off to TWO decimal places).

Q.33	If the ordinary differential equation $$x^2\frac{d^2\phi}{dx^2}+x\frac{d\phi}{dx}+x^2\phi=0,\ x>0$$ has a solution of the form $\phi(x)=x^r\sum_{n=0}^{\infty}a_nx^n$, where a_n's are constants and $a_0\neq 0$, then the value of r^2+1 is ________.

Q.34	The Bessel functions $J_\alpha(x),\ x > 0,\ \alpha \in \mathbb{R}$ satisfy $J_{\alpha-1}(x) + J_{\alpha+1}(x) = \frac{2\alpha}{x} J_\alpha(x)$. Then, the value of $(\pi J_{\frac{3}{2}}(\pi))^2$ is __________.

Q.35	The partial differential equation $$7\frac{\partial^2 u}{\partial x^2} + 16\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$$ is transformed to $$A\frac{\partial^2 u}{\partial \xi^2} + B\frac{\partial^2 u}{\partial \xi \partial \eta} + C\frac{\partial^2 u}{\partial \eta^2} = 0,$$ using $\xi = y - 2x$ and $\eta = 7y - 2x$. Then, the value of $\frac{1}{12^3}(B^2 - 4AC)$ is __________.

Q.36 – Q.65 Carry TWO marks each.

Q.36	Let $\mathbb{R}[X]$ denote the ring of polynomials in X with real coefficients. Then, the quotient ring $\mathbb{R}[X]/(X^4+4)$ is
(A)	a field
(B)	an integral domain, but not a field
(C)	not an integral domain, but has 0 as the only nilpotent element
(D)	a ring which contains non-zero nilpotent elements

Q.37	Consider the following conditions on two proper non-zero ideals J_1 and J_2 of a non-zero commutative ring R. **P:** For any $r_1, r_2 \in R$, there exists a unique $r \in R$ such that $r - r_1 \in J_1$ and $r - r_2 \in J_2$. **Q:** $J_1 + J_2 = R$ Then, which of the following statements is TRUE?
(A)	**P** implies **Q** but **Q** does not imply **P**
(B)	**Q** implies **P** but **P** does not imply **Q**
(C)	**P** implies **Q** and **Q** implies **P**
(D)	**P** does not imply **Q** and **Q** does not imply **P**

Q.38	Let $f : [-\pi, \pi] \to \mathbb{R}$ be a continuous function such that $f(x) > \frac{f(0)}{2}$, $\lvert x\rvert < \delta$ for some δ satisfying $0 < \delta < \pi$. Define $P_{n,\delta}(x) = (1 + \cos x - \cos\delta)^n$, for $n = 1, 2, 3, \cdots$. Then, which of the following statements is TRUE?
(A)	$\lim_{n\to\infty} \int_0^{2\delta} f(x)P_{n,\delta}(x)dx = 0$
(B)	$\lim_{n\to\infty} \int_{-2\delta}^{0} f(x)P_{n,\delta}(x)dx = 0$
(C)	$\lim_{n\to\infty} \int_{-\delta}^{\delta} f(x)P_{n,\delta}(x)dx = 0$
(D)	$\lim_{n\to\infty} \int_{[-\pi,\pi]\setminus[-\delta,\delta]} f(x)P_{n,\delta}(x)dx = 0$

Q.39	**P** : Suppose that $\sum_{n=0}^{\infty} a_n x^n$ converges at $x = -3$ and diverges at $x = 6$. Then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges. **Q**: The interval of convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \log_e n}$ is $[-4, 4]$. Which of the following statements is TRUE?
(A)	**P** is true and **Q** is true
(B)	**P** is false and **Q** is false
(C)	**P** is true and **Q** is false
(D)	**P** is false and **Q** is true

Q.40	Let $$f_n(x) = \frac{x^2}{x^2 + (1 - nx)^2}, \; x \in [0, 1], \; n = 1, 2, 3, \cdots .$$ Then, which of the following statements is TRUE?
(A)	$\{f_n\}$ is not equicontinuous on $[0, 1]$
(B)	$\{f_n\}$ is uniformly convergent on $[0, 1]$
(C)	$\{f_n\}$ is equicontinuous on $[0, 1]$
(D)	$\{f_n\}$ is uniformly bounded and has a subsequence converging uniformly on $[0, 1]$

Q.41	Let $(\mathbb{Q}, d)$ be the metric space with $d(x, y) = \lvert x - y \rvert$. Let $E = \{p \in \mathbb{Q} : 2 < p^2 < 3\}$. Then, the set E is
(A)	closed but not compact
(B)	not closed but compact
(C)	compact
(D)	neither closed nor compact

Q.42	Let $T : L^2[-1,1] \to L^2[-1,1]$ be defined by $Tf = \tilde{f}$, where $\tilde{f}(x) = f(-x)$ almost everywhere. If M is the kernel of $I - T$, then the distance between the function $\phi(t) = e^t$ and M is
(A)	$\frac{1}{2}\sqrt{(e^2 - e^{-2} + 4)}$
(B)	$\frac{1}{2}\sqrt{(e^2 - e^{-2} - 2)}$
(C)	$\frac{1}{2}\sqrt{(e^2 - 4)}$
(D)	$\frac{1}{2}\sqrt{(e^2 - e^{-2} - 4)}$

Q.43	Let X, Y and Z be Banach spaces. Suppose that $T : X \to Y$ is linear and $S : Y \to Z$ is linear, bounded and injective. In addition, if $S \circ T : X \to Z$ is bounded, then, which of the following statements is TRUE?
(A)	T is surjective
(B)	T is bounded but not continuous
(C)	T is bounded
(D)	T is not bounded

Q.44	The first derivative of a function $f \in C^{\infty}(-3,3)$ is approximated by an interpolating polynomial of degree 2, using the data $$(-1, f(-1)),\ (0, f(0)) \text{ and } (2, f(2)).$$ It is found that $$f'(0) \approx -\frac{2}{3}f(-1) + \alpha f(0) + \beta f(2).$$ Then, the value of $\dfrac{1}{\alpha\beta}$ is
(A)	3
(B)	6
(C)	9
(D)	12

Q.45	The work done by the force $F = (x+y)\hat{i} - (x^2+y^2)\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in $\overrightarrow{OX}$ and $\overrightarrow{OY}$ directions, respectively, along the upper half of the circle $x^2+y^2=1$ from $(1,0)$ to $(-1,0)$ in the xy-plane is
(A)	$-\pi$
(B)	$-\frac{\pi}{2}$
(C)	$\frac{\pi}{2}$
(D)	π

Q.46	Let $u(x,t)$ be the solution of the wave equation $$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0,\ 0 < x < \pi,\ t > 0,$$ with the initial conditions $$u(x,0) = \sin x + \sin 2x + \sin 3x,\ \frac{\partial u}{\partial t}(x,0) = 0,\ 0 < x < \pi$$ and the boundary conditions $u(0,t) = u(\pi,t) = 0,\ t \geq 0$. Then, the value of $u\left(\frac{\pi}{2}, \pi\right)$ is
(A)	$-1/2$
(B)	0
(C)	$1/2$
(D)	1

Q.47	Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $$T((1,2)) = (1,0) \text{ and } \mathrm{T}((2,1)) = (1,1).$$ For $p, q \in \mathbb{R}$, let $T^{-1}((p,q)) = (x,y)$. Which of the following statements is TRUE?
(A)	$x = p - q;\ y = 2p - q$
(B)	$x = p + q;\ y = 2p - q$
(C)	$x = p + q;\ y = 2p + q$
(D)	$x = p - q;\ y = 2p + q$

Q.48	Let $y = (\alpha, -1)^T,\ \alpha \in \mathbb{R}$ be a feasible solution for the dual problem of the linear programming problem $$\begin{aligned} \text{Maximize:} \quad & 5x_1 + 12x_2 \\ \text{subject to:} \quad & x_1 + 2x_2 + x_3 \leq 10 \\ & 2x_1 - x_2 + 3x_3 = 8 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$ Which of the following statements is TRUE?
(A)	$\alpha < 3$
(B)	$3 \leq \alpha < 5.5$
(C)	$5.5 \leq \alpha < 7$
(D)	$\alpha \geq 7$

Q.49	Let K denote the subset of $\mathbb{C}$ consisting of elements algebraic over $\mathbb{Q}$. Then, which of the following statements are TRUE?
(A)	No element of $\mathbb{C}\backslash K$ is algebraic over $\mathbb{Q}$
(B)	K is an algebraically closed field
(C)	For any bijective ring homomorphism $f : \mathbb{C} \longrightarrow \mathbb{C}$, we have $f(K) = K$
(D)	There is no bijection between K and $\mathbb{Q}$

Q.50	Let T be a Möbius transformation such that $T(0) = \alpha, T(\alpha) = 0$ and $T(\infty) = -\alpha$, where $\alpha = (-1+i)/\sqrt{2}$. Let L denote the straight line passing through the origin with slope -1, and let C denote the circle of unit radius centred at the origin. Then, which of the following statements are TRUE?
(A)	T maps L to a straight line
(B)	T maps L to a circle
(C)	T^{-1} maps C to a straight line
(D)	T^{-1} maps C to a circle

Q.51	Let $a > 0$. Define $D_a : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ by $(D_a f)(x) = \frac{1}{\sqrt{a}} f\left(\frac{x}{a}\right)$, almost everywhere, for $f \in L^2(\mathbb{R})$. Then, which of the following statements are TRUE?
(A)	D_a is a linear isometry
(B)	D_a is a bijection
(C)	$D_a \circ D_b = D_{a+b}$, $b > 0$
(D)	D_a is bounded from below

Q.52	Let $\{\phi_0, \phi_1, \phi_2, \cdots\}$ be an orthonormal set in $L^2[-1,1]$ such that $\phi_n = C_n P_n$, where C_n is a constant and P_n is the Legendre polynomial of degree n, for each $n \in \mathbb{N} \cup \{0\}$. Then, which of the following statements are TRUE?
(A)	$\phi_6(1) = 1$
(B)	$\phi_7(-1) = 1$
(C)	$\phi_7(1) = \sqrt{\frac{15}{2}}$
(D)	$\phi_6(-1) = \sqrt{\frac{13}{2}}$

Q.53	Let $X = (\mathbb{R}, T)$, where T is the smallest topology on $\mathbb{R}$ in which all the singleton sets are closed. Then, which of the following statements are TRUE?
(A)	$[0, 1)$ is compact in X
(B)	X is not first countable
(C)	X is second countable
(D)	X is first countable

Q.54	Consider $(\mathbb{Z}, T)$, where T is the topology generated by sets of the form $$A_{m,n} = \{m + nk \mid k \in \mathbb{Z}\},$$ for $m, n \in \mathbb{Z}$ and $n \neq 0$. Then, which of the following statements are TRUE?
(A)	$(\mathbb{Z}, T)$ is connected
(B)	Each $A_{m,n}$ is a closed subset of $(\mathbb{Z}, T)$
(C)	$(\mathbb{Z}, T)$ is Hausdorff
(D)	$(\mathbb{Z}, T)$ is metrizable

Q.55	Let $A \in \mathbb{R}^{m\times n}$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$. Consider the linear programming primal problem $$\begin{aligned} \text{Minimize:} \quad & c^T x \\ \text{subject to:} \quad & Ax = b \\ & x \geq 0. \end{aligned}$$ Let x^0 and y^0 be feasible solutions of the primal and its dual, respectively. Which of the following statements are TRUE?
(A)	$c^T x^0 \geq b^T y^0$
(B)	$c^T x^0 = b^T y^0$
(C)	If $c^T x^0 = b^T y^0$, then x^0 is optimal for the primal
(D)	If $c^T x^0 = b^T y^0$, then y^0 is optimal for the dual

Q.56	Consider $\mathbb{R}^3$ as a vector space with the usual operations of vector addition and scalar multiplication. Let $x \in \mathbb{R}^3$ be denoted by $x = (x_1, x_2, x_3)$. Define subspaces W_1 and W_2 by $$W_1 := \{x \in \mathbb{R}^3 : x_1 + 2x_2 - x_3 = 0\}$$ and $$W_2 := \{x \in \mathbb{R}^3 : 2x_1 + 3x_3 = 0\}.$$ Let dim(U) denote the dimension of the subspace U. Which of the following statements are TRUE?
(A)	$\dim(\mathrm{W}_1) = \dim(\mathrm{W}_2)$
(B)	$\dim(\mathrm{W}_1) + \dim(\mathrm{W}_2) - \dim(\mathbb{R}^3) = 1$
(C)	$\dim(\mathrm{W}_1 + \mathrm{W}_2) = 2$
(D)	$\dim(\mathrm{W}_1 \cap \mathrm{W}_2) = 1$

Q.57 Three companies C_1, C_2 and C_3 submit bids for three jobs J_1, J_2 and J_3. The costs involved per unit are given in the table below:

	J_1	J_2	J_3
C_1	10	12	8
C_2	9	15	10
C_3	15	10	9

Then, the cost of the optimal assignment is ____________.

Q.58	The initial value problem $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$ is solved by using the following second order Runge-Kutta method: $$\begin{aligned} K_1 &= hf(x_i,\ y_i) \\ K_2 &= hf(x_i + \alpha h,\ y_i + \beta K_1) \\ y_{i+1} &= y_i + \frac{1}{4}(K_1 + 3K_2),\quad i \geq 0, \end{aligned}$$ where h is the uniform step length between the points $x_0, x_1, \cdots, x_n$ and $y_i = y(x_i)$. The value of the product $\alpha\beta$ is ________ (round off to TWO decimal places).

Q.59	The surface area of the paraboloid $z = x^2 + y^2$ between the planes $z = 0$ and $z = 1$ is ____________ (round off to ONE decimal place).

Q.60	The rate of change of $f(x, y, z) = x + x\cos z - y\sin z + y$ at P_0 in the direction from $P_0(2, -1, 0)$ to $P_1(0, 1, 2)$ is ______________.

Q.61	If the Laplace equation $$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ 1<x<2,\ 1<y<2$$ with the boundary conditions $$\frac{\partial u}{\partial x}(1,y)=y,\ \frac{\partial u}{\partial x}(2,y)=5,\ 1<y<2$$ and $$\frac{\partial u}{\partial y}(x,1)=\frac{\alpha x^2}{7},\ \frac{\partial u}{\partial y}(x,2)=x,\ 1<x<2$$ has a solution, then the constant α is ________.

Q.62	Let $u(x, y)$ be the solution of the first order partial differential equation $$x\frac{\partial u}{\partial x} + (x^2 + y)\frac{\partial u}{\partial y} = u, \text{ for all } x, y \in \mathbb{R}$$ satisfying $u(2, y) = y - 4,\ y \in \mathbb{R}$. Then, the value of $u(1, 2)$ is ________.

Q.63	The optimal value for the linear programming problem $$\begin{aligned} \text{Maximize:} \quad & 6x_1 + 5x_2 \\ \text{subject to:} \quad & 3x_1 + 2x_2 \leq 12 \\ & -x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$ is ____________.

Q.64 A certain product is manufactured by plants P_1, P_2 and P_3 whose capacities are $15, 25$ and 10 units, respectively. The product is shipped to markets M_1, M_2, M_3 and M_4, whose requirements are $10, 10, 10$ and 20, respectively. The transportation costs per unit are given in the table below.

	M_1	M_2	M_3	M_4	
P_1	1	3	1	3	15
P_2	2	2	4	1	25
P_3	2	1	1	2	10
	10	10	10	20	

Then the cost corresponding to the starting basic solution by the Northwest-corner method is ____________.

Q.65	Let M be a 3×3 real matrix such that $M^2 = 2M + 3I$. If the determinant of M is -9, then the trace of M equals ______________.