COMPLEX ANALYSIS (PYPS)

JUNE – 2014

PART – B

1. Let f, g be meromorphic functions on $\mathbb{C}$. If f has a zero of order k at z=a and g has a pole of order m at z = 0, then g(f(z)) has
1. a zero of order km at z=a
2. a pole of order km at z=a
3. a zero of order |k-m| at z=a
4. a pole of order |k-m| at z=a

2. Let p(x) be a polynomial of the real variable x of degree $k \geq 1$. Consider the power series f(z) = $\sum_{n=0}^{\infty} p(n) z^n$, where z is a complex variable. Then the radius of convergence of f(z) is
1. 0
2. 1
3. k
4. ∞

PART – C

3. Let f be an entire function. Suppose, for each $a \in \mathbb{R}$, there exists at least one coefficient c_n in f(z) = $\sum_{n=0}^{\infty} c_n (z-a)^n$, which is zero. Then
1. $f^{(n)}(0) = 0$ for infinitely many $n \geq 0$
2. $f^{(2n)}(0) = 0$ for every n ≥ 0.
3. $f^{(2n+1)}(0) = 0$ for every $n \geq 0$
4. there exists k ≥ 0 such that $f^{(n)}(0) = 0$ for all n ≥ k.

4. Let $K \subseteq \mathbb{C}$ be a bounded set. Let H($\mathbb{C}$) denote the set of all entire functions and let $\mathbb{C}$(K) denote the set of all continuous functions on K. Consider the restriction map r:H($\mathbb{C}$) → $\mathbb{C}$(K) given by r(f)=$f_{|K}$. Then r is injective if
1. K is compact.
2. K is connected.
3. K is uncountable.
4. K is finite.

5. For $z \in \mathbb{C}$, define $f(z) = \frac{e^z}{e^z - 1}$. Then
1. f is entire.
2. the only singularities of f are poles.
3. f has infinitely many poles on the imaginary axis.
4. each pole of f is simple.

6. Let $D = \{z \in \mathbb{C} : |z| < 1\}$. Then there exists a holomorphic function $f: D \to \overline{D}$ with f(0)=0 with the property
1. $f'(0) = \frac{1}{2}$
2. $\left| f\left(\frac{1}{3}\right) \right| = \frac{1}{4}$
3. $f\left(\frac{1}{3}\right) = \frac{1}{2}$
4. $|f'(0)| = \sec\left(\frac{\pi}{6}\right)$

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PART – B

7. Let $p(z) = a_0 + a_1 z + ... + a_n z^n$ and $q(z) = b_1 z + b_2 z^2 + ... + b_n z^n$ be complex polynomials. If a_0, b_1 are non-zero complex numbers then the residue of p(z)/q(z) at 0 is equal to
1. $\frac{a_0}{b_1}$
2. $\frac{b_1}{a_0}$
3. $\frac{a_1}{b_1}$
4. $\frac{a_0}{a_1}$

8. Let $\sum_{n=1}^{\infty} a_n z^n$ be a convergent power series such that $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = R > 0$. Let p be a polynomial of degree d. Then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n p(n) z^n$ equals
1. R
2. d
3. Rd
4. R+d

9. Let f be an entire function on $\mathbb{C}$ and let Ω be a bounded open subset of $\mathbb{C}$. Let $S = \{\text{Re } f(z) + \text{Im } f(z) \mid z \in \Omega\}$. Which of the following statements is/are necessarily correct?
1. S is an open set in $\mathbb{R}$
2. S is a closed set in $\mathbb{R}$
3. S is an open set of $\mathbb{C}$
4. S is a discrete set in $\mathbb{R}$

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10. Let $u(x+iy)=x^3-3xy^2+2x$. For which of the following functions v, is u+iv a holomorphic function on $\mathbb{C}$?

1. $v(x+iy)=y^3-3x^2y+2y$
2. $v(x+iy)=3x^2y-y^3+2y$
3. $v(x+iy)=x^3-3xy^2+2x$
4. $v(x+iy)=0$

PART – C

11. Let f be an entire function on $\mathbb{C}$. Let $g(z)=\overline{f(\bar{z})}$. Which of the following statements is/are correct?

1. if $f(z)\in\mathbb{R}$ for all $z\in\mathbb{R}$ then f=g
2. if $f(z)\in\mathbb{R}$ for all $z\in\{z\,|\,\mathrm{Im}\,z=0\}\cup\{z\,|\,\mathrm{Im}\,z=a\}$, for some a>0, then $f(z+ia)=f(z-ia)$ for all $z\in\mathbb{C}$.
3. If $f(z)\in\mathbb{R}$ for all $z\in\{z\,|\,\mathrm{Im}\,z=0\}\cup\{z\,|\,\mathrm{Im}\,z=a\}$, for some a>0, then $f(z+2ia)=f(z)$ for all $z\in\mathbb{C}$
4. If $f(z)\in\mathbb{R}$ for all $z\in\{z\,|\,\mathrm{Im}\,z=0\}\cup\{z\,|\,\mathrm{Im}\,z=a\}$ for some a > 0, then $f(z+ia)=f(z)$ all $z\in\mathbb{C}$

12. Let $f(z)=\sum_{n=0}^{\infty}a_nz^n$ be an entire function and let r be a positive real number. Then

1. $\sum_{n=0}^{\infty}|a_n|^2 r^{2n}\le \sup_{|z|=r}|f(z)|^2$
2. $\sup_{|z|=r}|f(z)|^2\le\sum_{n=0}^{\infty}|a_n|^2 r^{2n}$
3. $\sum_{n=0}^{\infty}|a_n|^2 r^{2n}\le\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^2\,d\theta$
4. $\sup_{|z|=r}|f(z)|^2\le\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^2\,d\theta$

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PART – B

13. $\int_{|z+1|=2}\frac{z^2}{4-z^2}dz=$

1. 0	2. $-2\pi i$
3. $2\pi i$	4. 1

14. How many elements does the set $\{z\in C/z^{60}=-1, z^k\neq-1 \text{ for } 0<k<60\}$ have?

1. 24	2. 30
3. 32	4. 45

15. Let f be a real valued harmonic function on $\mathbb{C}$, that is, f satisfies the equation $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$.

Define the functions
$$g=\frac{\partial f}{\partial x}-i\frac{\partial f}{\partial y},\quad h=\frac{\partial f}{\partial x}+i\frac{\partial f}{\partial y}$$
Then

1. g and h are both holomorphic functions.
2. g is holomorphic, but h need not be holomorphic.
3. h is holomorphic, but g need not be holomorphic.
4. both g and h are identically equal to the zero function.

PART – C

16. Let f be an entire function. Which of the following statements are correct?

1. f is constant if the range of f is contained in a straight line.
2. f is constant if f has uncountably many zeros.
3. f is constant if f is bounded on {z∈$\mathbb{C}$: Re(z) ≤ 0}.
4. f is constant if the real part of f is bounded.

17. Let f be analytic function defined on the open unit disc in $\mathbb{C}$. Then f is constant if

1. $f\left(\frac{1}{n}\right)=0$ for all $n\ge1$.
2. $f(z)=0$ for all $|z|=\frac{1}{2}$.
3. $f\left(\frac{1}{n^2}\right)=0$ for all $n\ge1$.
4. $f(z)=0$ for all $z\in(-1,1)$.

18. Let p be a polynomial in 1-complex variable. Suppose all zeroes of p are in the upper half plane $H=\{z\in C\,|\,\mathrm{Im}(z)>0\}$. Then

1. $\mathrm{Im}\frac{p'(z)}{p(z)}>0$ for $z\in\mathbb{R}$.

2. $\operatorname{Re} i\dfrac{p'(z)}{p(z)} < 0 \ \ for\, z \in \mathbb{R}$

3. $\operatorname{Im}\dfrac{p'(z)}{p(z)} > 0 \ \ for\, z \in \mathbb{C},\ with\ \operatorname{Im} z < 0$

4. $\operatorname{Im}\dfrac{p'(z)}{p(z)} > 0 \ \ for\, z \in \mathbb{C}\, with\ \operatorname{Im} z > 0$

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PART – B

19. Let a, b, c, d ∈ ℝ be such that ad – bc > 0. Consider the Mobius transformation

$T_{a,b,c,d}(z) = \dfrac{az+b}{cz+d}$. Define

$H_+ = \{z \in \mathbb{C} : \text{Im}(z) > 0\}$, $H_- = \{z \in \mathbb{C} : \text{Im}(z) < 0\}$

$R_+ = \{z \in \mathbb{C} : \text{Re}(z) > 0\}$, $R_- = \{z \in \mathbb{C} : \text{Re}(z) < 0\}$.

Then, $T_{a,b,c,d}$ maps

1. H_+ to H_+.
2. H_+ to H_-
3. R_+ to R_+.
4. R_+ to R_-

20. What is the cardinality of the set

$\{z \in \mathbb{C} \mid z^{98} = 1 \text{ and } z^n \neq 1 \text{ for any } 0 < n < 98\}$?

1. 0.
2. 12.
3. 42.
4. 49.

21. Consider the following power series in the complex variable z:

$f(z) = \sum_{n=1}^{\infty} n \log n\, z^n, g(z) = \sum_{n=1}^{\infty} \dfrac{e^{n^2}}{n} z^n$. If r, R are the radii of convergence of f and g respectively, then

1. r = 0, R = 1.
2. r = 1, R = 0.
3. r = 1, R = ∞.
4. r = ∞, R = 1.

PART – C

22. Let $f(z) = \dfrac{1}{e^z - 1}$ for all z ∈ ℂ such that $e^z \neq 1$. Then

1. f is meromorphic.
2. the only singularities of f are poles.
3. f has infinitely many poles on the imaginary axis.
4. Each pole of f is simple.

23. Let f be an analytic function in ℂ. Then f is constant if the zero set of f contains the sequence

1. $a_n = 1/n$
2. $a_n = (-1)^{n-1}\dfrac{1}{n}$
3. $a_n = \dfrac{1}{2n}$
4. $a_n = n$ if 4 does not divide n and $a_n = \dfrac{1}{n}$ if 4 divides n

24. Consider the function $f(z) = \dfrac{1}{z}$ on the annulus $A = \left\{z \in C : \dfrac{1}{2} < |z| < 2\right\}$.

Which of the following is/are true?

1. There is a sequence $\{p_n(z)\}$ of polynomials that approximate f(z) uniformly on compact subsets of A.
2. There is a sequence $\{r_n(z)\}$ of rational functions, whose poles are contained in ℂ\A and which approximates f(z) uniformly on compact subsets of A.
3. No sequence $\{p_n(z)\}$ of polynomials approximate f(z) uniformly on compact subsets of A.
4. No sequence $\{r_n(z)\}$ of rational functions whose poles are contained in ℂ\A, approximate f(z) uniformly on compact subsets of A.

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PART – B

25. Let P(z), Q(z) be two complex non-constant polynomials of degree m,n respectively. The number of roots of P(z)=P(z)Q(z) counted with multiplicity is equal to:

1. min {m,n}
2. max {m,n}
3. m+n
4. m-n

26. Let D be the open unit disc in C and H(D) be the collection of all holomorphic functions on it.

Let $S = \left\{ f \in H(D) : f\left(\dfrac{1}{2}\right) = \dfrac{1}{2}, f\left(\dfrac{1}{4}\right) = \dfrac{1}{4}, \ldots, f\left(\dfrac{1}{2n}\right) = \dfrac{1}{2n}, \ldots \right\}$ and

$T = \left\{ f \in H(D) : f\left(\dfrac{1}{2}\right) = f\left(\dfrac{1}{3}\right) = \dfrac{1}{2}, f\left(\dfrac{1}{4}\right) = f\left(\dfrac{1}{5}\right) = \dfrac{1}{4}, \ldots, f\left(\dfrac{1}{2n}\right) = f\left(\dfrac{1}{2n+1}\right) = \dfrac{1}{2n}, \ldots \right\}$ Then

1. Both S,T are singleton sets
2. S is a singleton set but T= ϕ
3. T is a singleton set but S= ϕ

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4. Both S,T are empty

27. Let P(x) be a polynomial of degree $d \geq 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty} p(n)z^n$ is

1. 0 2. 1
3. ∞ 4. dependent on d

28. The residue of the function $f(z) = e^{-e^{1/z}}$ at z=0 is:

1. $1+e^{-1}$ 2. e^{-1}
3. $-e^{-1}$ 4. $1-e^{-1}$

PART – C

29. Let H = {z = x + iy $\in \mathbb{C}$: y > 0} be the upper half plane and D = {z $\in \mathbb{C}$: |z| < 1} be the open unit disc. Suppose that f is a Mobius transformation, which maps H conformally onto D. Suppose that f(2i) = 0. Pick each correct statement from below.

1. f has a simple pole at z = -2i.
2. f satisfies $f(i)\,\overline{f(-i)} = 1$.
3. f has an essential singularity at z = - 2i.
4. $|f(2+2i)| = \frac{1}{\sqrt{5}}$.

30. Consider the function $F(z) = \int_1^2 \frac{1}{(x-z)^2}dx,$ Im(z) > 0. Then there is a meromorphic function G(z) on $\mathbb{C}$ that agrees with F(z) when Im (z) > 0, such that

1. 1, ∞ are poles of G(z)
2. 0,1,∞ are poles of G(z)
3. 1,2 are poles of G(z)
4. 1,2 are simple poles of G(z).

31. Let f : $\mathbb{C} \to \mathbb{C}$ be an entire function. Suppose that f = u + iv where u,v are the real and imaginary parts of f respectively. Then f is constant if

1. {u(x,y) : z = x + iy$\in\mathbb{C}$} is bounded
2. {v(x,y) : z = x + iy$\in\mathbb{C}$} is bounded
3. {u(x,y) +v(x,y):z = x + iy$\in\mathbb{C}$} is bounded
4. {u^2(x,y) +v^2(x,y):z =x + iy$\in\mathbb{C}$} is bounded

32. Let A = $\{z\in\mathbb{C} \mid |z| > 1\}$, B = $\{z\in\mathbb{C} \mid z \neq 0\}$. Which of the following are true?

1. There is a continuous onto function f:A $\to$ B
2. There is a continuous one to one function f : B $\to$ A
3. There is a nonconstant analytic function f : B $\to$ A
4. There is a nonconstant analytic function f : A $\to$ B

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PART - B

33. The radius of convergence of the series $\sum_{n=1}^{\infty} z^{n^2}$ is

1. 0 2. ∞
3. 1 4. 2

34. Let C be the circle |z| = 3/2 in the complex plane that is oriented in the counter clockwise direction. The value of a for which $\int_C \left(\frac{z+1}{z^2-3z+2} + \frac{a}{z-1} \right) dz = 0$ is

1. 1 2. -1
3. 2 4. -2

35. Suppose f and g are entire functions and g(z) $\neq$ 0 for all z $\in \mathbb{C}$. If |f(z)| $\leq$ |g(z)|, then we conclude that

1. f(z) $\neq$ 0 for all z $\in \mathbb{C}$.
2. f is a constant function.
3. f(0) = 0.
4. for some C $\in \mathbb{C}$, f(z) = Cg(z).

36. Let f be a holomorphic function on 0 < |z| < ϵ, ϵ > 0 given by a convergent Laurent series $\sum_{n=-\infty}^{\infty} a_n z^n$. Given also that $\lim_{z\to 0}|f(z)| = \infty$, We can conclude that

1. $a_{-1} \neq 0$ and $a_{-n} = 0$ for all n $\geq$ 2
2. $a_{-N} \neq 0$ for some N $\geq$ 1 and a_{-n} = 0 for all n > N
3. a_{-n} = 0 for all n $\geq$ 1
4. $a_{-n} \neq 0$ for all n $\geq$ 1

PART - C

37. Let f(z) be the meromorphic function given by $\frac{z}{(1-e^z)\sin z}$. Then

1. z=0 is a pole of order 2.

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2. for every k∈$\mathbb{Z}$, z=2πik is a simple pole.
3. for every k∈$\mathbb{Z}\setminus\{0\}$, z=kπ is a simple pole.
4. z=π+2πi is a pole.

38. Consider the polynomial

$P(z)=\sum_{n=1}^{N} a_n z^n, 1\le N<\infty,\ a_n\in\mathbb{R}\setminus\{0\}$. Then, with **D** = {w∈$\mathbb{C}$: |w| < 1}

1. P(D) ⊆ $\mathbb{R}$ 2. P(D) is open
3. P(D) is closed 4. P(D) is bounded

39. Consider the polynomial

$P(z)=\left(\sum_{n=0}^{5} a_n z^n\right)\left(\sum_{n=0}^{9} b_n z^n\right)$ where $a_n, b_n\in\mathbb{R}$

∀ n, $a_5 \neq 0$, $b_9 \neq 0$. Then counting roots with multiplicity we can conclude that P(z) has
1. at least two real roots.
2. 14 complex roots
3. no real roots
4. 12 complex roots.

40. Let **D** be the open unit disc in $\mathbb{C}$. Let g: **D** →**D** be holomorphic, g(0)=0, and let

$h(z)=\begin{cases}\frac{g(z)}{z}, & z\in D, z\neq 0\\ g'(0), & z=0\end{cases}$. Which of the following statements are true?

1. h is holomorphic in **D**. 2. h(**D**) ⊆ $\overline{\mathbf{D}}$.
3. $|g'(0)|>1$ 4. $\left|g\left(\frac{1}{2}\right)\right|\le\frac{1}{2}$

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PART – B

41. Let C denote the unit circle centered at the origin in $\mathbb{C}$. Then $\frac{1}{2\pi i}\int_C |1+z+z^2|^2\, dz$, where the integral is taken anti-clockwise along C, equals
1. 0 2. 1
3. 2 4. 3

42. Consider the power series

$f(x)=\sum_{n=2}^{\infty}\log(n)x^n.$

The radius of convergence of the series $f(x)$ is
1. 0 2. 1 3. 3 4. ∞

43. For an odd integer k ≥ 1, let $\mathcal{F}$ be the set of all entire functions f such that

$f(x)=|x^k|$ for all $x\in(-1,1)$. Then the cardinality of $\mathcal{F}$ is
1. 0
2. 1
3. strictly greater than 1 but finite
4. infinite

44. Suppose f is holomorphic in an open neighbourhood of $z_0\in\mathbb{C}$. Given that the series $\sum_{n=0}^{\infty} f^{(n)}(z_0)$ converges absolutely, we can conclude that
1. f is constant
2. f is a polynomial
3. f can be extended to an entire function
4. $f(x)\in\mathbb{R}$ for all $x\in\mathbb{R}$

PART - C

45. Let $f=u+iv$ be an entire function where u,v are the real and imaginary parts of f respectively. If the Jacobian matrix $J_a=\begin{bmatrix}u_x(a) & u_y(a)\\ v_x(a) & v_y(a)\end{bmatrix}$ is symmetric for all a∈$\mathbb{C}$, then
1. f is a polynomial.
2. f is a polynomial of degree ≤1.
3. f is necessarily a constant function
4. f is a polynomial of degree strictly greater than 1.

46. Consider the function $f(z)=\frac{\sin(\pi z/2)}{\sin(\pi z)}$.

Then f has poles at
1. all integers
2. all even integers
3. all odd integers
4. all integers of the form 4k+1, k∈$\mathbb{Z}$

47. Consider the Möbius transformation

$f(z)=\frac{1}{z}, z\in\mathbb{C}$, z≠0. If C denotes a circle with positive radius passing through the origin, then f map C\{0} to
1. a circle
2. a line
3. a line passing through the origin
4. a line not passing through the origin

48. For which among the following functions f(z) defined on G=$\mathbb{C}\setminus\{0\}$, is there no sequence of polynomials approximating f(z) uniformly on compact subsets of G?
1. exp(z) 2. 1/z
3. z^2 4. $1/z^2$

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PART – B

49. The function f : $\mathbb{C} \to \mathbb{C}$ defined by
$f(z) = e^z + e^{-z}$ has
1. finitely many zeros
2. no zeros
3. only real zeros
4. has infinitely many zeros

50. Let f be a holomorphic function in the open unit disc such that $\lim_{z\to 1} f(z)$ does not exist. Let $\sum_{n=0}^{\infty} a_n z^n$ be the Taylor series of f about z = 0 and let R be its radius of convergence. Then
1. R = 0 2. 0 < R < 1
3. R = 1 4. R > 1

51. Let C be the circle of radius 2 with centre at the origin in the complex plane, oriented in the anti-clockwise direction. Then the integral
$\oint_C \frac{dz}{(z-1)^2}$ is equal to
1. $\frac{1}{2\pi i}$ 2. $2\pi i$
3. 1 4. 0

52. Let **D** be the open unit disc in the complex plane and $U = \mathbf{D} \setminus \left\{-\frac{1}{2}, \frac{1}{2}\right\}$. Also, let
H_1 = {f : **D**→$\mathbb{C}$ | f is holomorphic and bounded} and H_2 = {f : **U** → $\mathbb{C}$ | f is holomorphic and bounded}. Then the map r : $H_1 \to H_2$ given by r(f) = $f|_U$, the restriction of f to U, is
1. injective but not surjective
2. surjective but not injective
3. injective and surjective
4. neither injective nor surjective

PART – C

53. Let f be an entire function. Consider A={z∈$\mathbb{C}$|$f^{(n)}(z)$=0 for some positive integer n}. Then
1. if A = $\mathbb{C}$, then f is a polynomial
2. if A = $\mathbb{C}$, then f is a constant function
3. if A is uncountable, then f is a polynomial
4. If A is uncountable, then f is a constant function

54. Let f:$\mathbb{C}\to\mathbb{C}$ be a holomorphic function and let u be the real part of f and v the imaginary part of f. Then, for x,y∈$\mathbb{R}$. $|f'(x+iy)|^2$ it equal to
1. $u_x^2 + u_y^2$ 2. $u_x^2 + v_x^2$
3. $v_y^2 + u_y^2$ 4. $v_y^2 + v_x^2$

55. Let $p(z) = z^n + a_{n-1} z^{n-1} + ... + a_0$, where $a_0, ..., a_{n-1}$ are complex numbers and let
$q(z) = 1 + a_{n-1} z + ... + a_0 z^n$. If $|p(z)| \le 1$ for all z with $|z| \le 1$ then
1. $|q(z)| \le 1$ for all z with $|z| \le 1$
2. q(z) is a constant polynomial
3. $p(z) = z^n$ for all complex numbers z
4. p(z) is a constant polynomial

56. Let f be a non-constant entire function and let E be the image of f. Then
1. E is an open set
2. E ∩ {z:|z|<1} is empty
3. E ∩$\mathbb{R}$ is non – empty
4. E is a bounded set

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PART – B

57. The value of the integral $\oint_{|1-z|=1} \frac{e^z}{z^2-1} dz$ is
1. 0 2. $(\pi i)e$
3. $(\pi i)e - (\pi i)e^{-1}$ 4. $(e + e^{-1})$

58. Let $f : \{z/|z|<1\} \to \mathbb{C}$ be a non-constant analytic function. Which of the following conditions can possibly be satisfied by f ?
1. $f\left(\frac{1}{n}\right) = f\left(\frac{-1}{n}\right) = \frac{1}{n^2}\ \forall\ n \in \mathbb{N}$
2. $f\left(\frac{1}{n}\right) = f\left(\frac{-1}{n}\right) = \frac{1}{2n+1}\ \forall\ n \in \mathbb{N}$
3. $\left|f\left(\frac{1}{n}\right)\right| < 2^{-n}\ \forall\ n \in \mathbb{N}$
4. $\frac{1}{\sqrt{n}} < \left|f\left(\frac{1}{n}\right)\right| < \frac{2}{\sqrt{n}}\ \forall\ n \in \mathbb{N}$

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59. Consider the map $\varphi:\mathbb{C}\backslash\{1\} \to \mathbb{C}$ given by $\varphi(z) = \frac{1+z}{1-z}$. Which of the following is true?

1. $\varphi(\{z \in \mathbb{C} \mid |z| < 1\}) \subseteq \{z\in\mathbb{C} \mid |z| < 1\}$
2. $\varphi(\{z \in \mathbb{C} \mid \mathrm{Re}(z) < 0\})\subseteq\{z\in\mathbb{C} \mid \mathrm{Re}(z) < 0\}$
3. φ is onto
4. $\varphi(\mathbb{C}\backslash \{1\}) = \mathbb{C}\backslash \{-1\}$

60. Suppose that f is a non constant analytic function defined over $\mathbb{C}$. Then which of the following is false?

1. f is unbounded
2. f sends open sets into open sets
3. there exists an open connected domain U on which f is never f|u attains its minimum at one point of u
4. the image of f is dense in $\mathbb{C}$

PART – C

61. Let Ω be an open connected subset of $\mathbb{C}$. Let $E = \{z_1, z_2, \ldots, z_r\} \subseteq \Omega$. Suppose that $f : \Omega \to \mathbb{C}$ is a function such that $f_{|(\Omega\backslash E)}$ is analytic. Then f is analytic on Ω if

1. f is continuous on Ω
2. f is bounded on Ω
3. for every j, if $\sum_{m\in Z} a_m(z-z_j)^m$ is Laurent series expansion of f at z_j, then $a_m = 0$ for m = -1, -2, -3, …
4. for every j, if $\sum_{m\in Z} a_m(z-z_j)^m$ is Laurent series expansion of f at z_j, then $a_{-1} = 0$

62. Suppose that f: $\mathbb{C} \to \mathbb{C}$ is an analytic function. Then f is a polynomial if

1. for any point a $\in \mathbb{C}$, if $f(z) = \sum_0^\infty a_n (z-a)^n$ is a power series expansion at a, then $a_n = 0$ for at least one n
2. $\lim_{|z|\to\infty} |f(z)| = \infty$
3. $\lim_{|z|\to\infty} |f(z)| = M$ for some M
4. $|f(z)| \le M|z|^n$ for |z| sufficiently large and for some n

63. Let **D** be the open unit disk centered at 0 in $\mathbb{C}$ and f: **D** $\to \mathbb{C}$ be an analytic function. Let f = u + iv, where u, v are the real and imaginary parts of f. If $f(z) = \sum a_n z^n$ is the power series of f, then f is constant if

1. $\bar{f}$ is analytic
2. $u(1/2) \ge u(z)\ \forall z \in$ **D**
3. The set $\{n \in \mathbb{N} \mid a_n = 0\}$ is infinite
4. For any closed curve γ in **D**, $\int_\gamma \frac{f(z)\,dz}{(z-a)^2} = 0\ \forall$ a $\in$ **D** with $|a| \ge \frac{1}{2}$

64. Which of the following statements are true?

1. If $\{a_k\}$ is bounded then $\sum_0^\infty a_k z^k$ defines an analytic function on the open unit disk
2. If $\sum_0^\infty a_k z^k$ defines an analytic function on the open disk then $\{a_k\}$ must converge to zero
3. If $f(z) = \sum_0^\infty a_k z^k$ and $g(z) = \sum_0^\infty b_k z^k$ are two power series functions whose radii of convergence are 1, then the product f.g has a power series representation of the form $\sum_0^\infty c_k z^k$ on the open unit disk
4. If $f(z) = \sum_0^\infty a_k z^k$ has a radius of convergence 1, then f is continuous on $\Omega = \{z \in\mathbb{C} \mid |z| \le 1\}$

DECEMBER – 2018

PART – B

65. Consider the polynomials p(z), q(z) in the complex variable z and let $I_{p,q} = \oint_\gamma p(z)\overline{q(z)}dz$, where γ denotes the closed contour $\gamma(t) = e^{it}, 0 \le t \le 2\pi$. Then

1. $I_{z^m,z^n} = 0$ for all positive integers m,n with m≠n
2. $I_{z^n,z^n} = 2\pi i$ for all positive integers n
3. $I_{p,1} = 0$ for all polynomials p
4. $I_{p,q} = p(0)\overline{q(0)}$ for all polynomials p,q

66. Let $\gamma(t) = 3e^{it}, 0 \le t \le 2\pi$ be the positively oriented circle of radius 3 centred at the origin. The value of λ for which $\oint_\gamma \frac{\lambda}{z-2}dz = \oint_\gamma \frac{1}{z^2-5z+4}dz$ is

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1. $\lambda=\frac{-1}{3}$ 2. $\lambda=0$
3. $\lambda=\frac{1}{3}$ 4. $\lambda=1$

67. Let f: $\mathbb{C}\to\mathbb{C}$ be a non-constant entire function and let Image (f) = {w∈$\mathbb{C}$:∃ z∈$\mathbb{C}$ such that f(z)=w}. Then
1. The interior of Image (f) is empty
2. Image (f) intersects every line passing through the origin
3. There exists a disc in the complex plane, which is disjoint from Image (f)
4. Image (f) contains all its limit points

PART - C

68. Let H denote the upper half plane, that is, H = {z = x + iy : y > 0}. For z ∈ H, which of the following are true?
1. $\frac{1}{z}\in H$
2. $\frac{1}{z^2}\in H$
3. $\frac{-z}{z+1}\in H$
4. $\frac{z}{2z+1}\in H$

69. Let f : $\mathbb{C}\to\mathbb{C}$ be an analytic function. Then which of the following statements are true?
1. If |f(z)| ≤ 1 for all z ∈ $\mathbb{C}$, then f ′ has infinitely many zeroes in $\mathbb{C}$
2. If f is onto, then the function f (cos z) is onto
3. If f is onto, then the function f(e^z) is onto
4. If f is one-one, then the function f(z^4 + z + 2) is one-one

70. Consider the entire functions f(z) = 1+z +z^{20} and g(z) = e^z, z∈$\mathbb{C}$. Which of the following statements are true ?
1. $\lim_{|z|\to\infty}|f(z)|=\infty$
2. $\lim_{|z|\to\infty}|g(z)|=\infty$
3. f^{-1}({z∈$\mathbb{C}$: |z|≤R}) is bounded for every R>0
4. g^{-1}({z∈$\mathbb{C}$: |z|≤R}) is bounded for every R>0

71. Which of the following statements are true?
1. tan z is an entire function
2. tan z is a meromorphic function on $\mathbb{C}$
3. tan z has an isolated singularity at ∞
4. tan z has a non-isolated singularity at ∞

JUNE – 2019

PART – B

72. Let C be the counter-clockwise oriented circle of radius $\frac{1}{2}$ centred at $i=\sqrt{-1}$. Then the value of the contour integral $\oint_C \frac{dx}{x^4-1}$ is
1. -π/2 2. π/2
3. -π 4. π

73. Consider the function f : $\mathbb{C}\to\mathbb{C}$ given by f(z) = e^z. Which of the following is false?
1. f({z ∈ $\mathbb{C}$: |z|< 1}) is not an open set
2. f({z ∈ $\mathbb{C}$: |z| ≤ 1}) is not an open set
3. f({z ∈ $\mathbb{C}$: |z|= 1}) is a closed set
4. f({z ∈ $\mathbb{C}$: |z| > 1}) is an unbounded open set

74. Given a real number a > 0, consider the triangle Δ with vertices 0, a, a + ia. If Δ is given the counter clockwise orientation, then the contour integral $\oint_\Delta \mathrm{Re}(z)\,dz$ (with Re (z) denoting the real part of z) is equal to
1. 0 2. $i\frac{a^2}{2}$
3. ia^2 4. $i\frac{3a^2}{2}$

75. Let f : $\mathbb{C}\to\mathbb{C}$ be an entire function such that $\lim_{x\to0}\left|f\left(\frac{1}{x}\right)\right|=\infty$. Then which of the following is true?
1. f is constant
2. f can have infinitely many zeros
3. f can have at most finitely many zeros
4. f is necessarily nowhere vanishing

PART – C

76. Let f(z) = (z^3 + 1) sin z^2 for z ∈ $\mathbb{C}$. Let f(z) = u (x, y) + i v (x, y), where z = x + iy and u, v are real valued functions. Then which of the following are true?

1. $u : \mathbb{R}^2 \to \mathbb{R}$ is infinitely differentiable
2. u is continuous but need not be differentiable
3. u is bounded
4. f can be represented by an absolutely convergent power series $\sum_{n=0}^{\infty} a_n z^n$ for all $z \in \mathbb{C}$

77. Let Re(z), Im(z) denote the real and imaginary parts of $z \in \mathbb{C}$, respectively. Consider the domain $\Omega = \{z \in \mathbb{C} : \text{Re}(z) > |\text{Im}(z)|\}$ and let $f_n(z) = \log z^n$, where $n \in \{1, 2, 3, 4\}$ and where $\log : \mathbb{C} \setminus (-\infty, 0] \to \mathbb{C}$ defines the principal branch of logarithm. Then which of the following are true?

1. $f_1(\Omega) = \{z \in \mathbb{C} : 0 \le |\text{Im}(z)| < \pi/4\}$
2. $f_2(\Omega) = \{z \in \mathbb{C} : 0 \le |\text{Im}(z)| < \pi/2\}$
3. $f_3(\Omega) = \{z \in \mathbb{C} : 0 \le |\text{Im}(z)| < 3\pi/4\}$
4. $f_4(\Omega) = \{z \in \mathbb{C} : 0 \le |\text{Im}(z)| < \pi\}$

78. Consider the set
$F = \{f : \mathbb{C} \to \mathbb{C} \mid f \text{ is an entire function}, |f'(z)| \le |f(z)| \text{ for all } z \in \mathbb{C}\}$.
Then which of the following are true?

1. F is a finite set
2. F is an infinite set
3. $F = \{\beta e^{\alpha z} : \beta \in \mathbb{C}, \alpha \in \mathbb{C}\}$
4. $F = \{\beta e^{\alpha z} : \beta \in \mathbb{C}, |\alpha| \le 1\}$

79. Let $D = \{z \in \mathbb{C} \mid |z| < 1\}$ and $\omega \in D$. Define $F_\omega : D \to D$ by $F_\omega(z) = \dfrac{\omega - z}{1 - \overline{\omega} z}$. Then which of the following are true?

1. F is one to one
2. F is not one to one
3. F is onto
4. F is not onto

DECEMBER – 2019

PART – B

80. For $z \in \mathbb{C}$, let $f(z) = \begin{cases} \dfrac{\overline{z}^2}{z} & \text{if } z \ne 0. \\ 0 & \text{otherwise.} \end{cases}$

Then which of the following statements is false?

1. f(z) is continuous everywhere
2. f(z) is not analytic in any open neighbourhood of zero
3. zf(z) satisfies the Cauchy-Riemann equations at zero
4. f(z) is analytic in some open subset of $\mathbb{C}$

81. Let $T : \mathbb{C} \to M_2(\mathbb{R})$ be the map given by
$$T(z) = T(x + iy) = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$$
Then which of the following statements is false?

1. $T(z_1 z_2) = T(z_1)\, T(z_2)$ for all $z_1, z_2 \in \mathbb{C}$
2. T(z) is singular if and only if z = 0
3. There does not exist non-zero $A \in M_2(\mathbb{R})$ such that the trace of T(z)A is zero for all $z \in \mathbb{C}$
4. $T(z_1 + z_2) = T(z_1) + T(z_2)$ for all $z_1, z_2 \in \mathbb{C}$

82. Consider the polynomial $f(z) = z^2 + az + p^{11}$, where $a \in Z\backslash\{0\}$ and $p \ge 13$ is a prime. Suppose that $a^2 \le 4p^{11}$. Which of the following statements is true?

1. f has a zero on the imaginary axis
2. f has a zero for which the real and imaginary parts are equal
3. f has distinct roots
4. f has exactly one real root

83. Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function with $f\left(\dfrac{1}{n}\right) = \dfrac{1}{n^2}$ for all $n \in \mathbb{N}$. Then which of the following statements is true?

1. No such f exists
2. such an f is not unique
3. $f(z) = z^2$ for $z \in \mathbb{C}$
4. f need not be a polynomial function

PART – C

84. Let U be an open subset of $\mathbb{C}$ and $f : U \to \mathbb{C}$ be an analytic function. Then which of the following are true?

1. If f is one-one, then f(U) is open in $\mathbb{C}$

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2. If f is onto, then U = $\mathbb{C}$
3. If f is onto, then f is one-one
4. f(U) is closed in $\mathbb{C}$, then f(U) is connected

85. Let f : $\mathbb{C} \to \mathbb{C}$ be an analytic function. For $z_0 \in \mathbb{C}$, which of the following statements are true?
1. can take the value z_0 at finitely many points in $\frac{1}{n} | n \in \mathbb{N}$
2. $f(1/n) = z_0$ for all $n \in \mathbb{N} \Rightarrow$ f is the constant function z_0
3. $f(n) = z_0$ for all $n \in \mathbb{N} \Rightarrow$ f is the constant function z_0
4. $f(r) = z_0$ for all $r \in \mathbb{Q} \cap [1, 2] \Rightarrow$ f is the constant function z_0

86. Let $U \subset \mathbb{C}$ be an open connected set and f: U $\to \mathbb{C}$ be a non-constant analytic function. Consider the following two sets:
X = {z $\in$ U : f(z) = 0}
Y = {z $\in$ U : f vanishes on an open neighbourhood of z in U}
Then which of the following statements are true?
1. X is closed in U
2. Y is closed in U
3. X has empty interior
4. Y is open in U

87. Consider the power series $f(z) = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n)!}$. Which of the following are true?
1. Radius of convergence of f(z) is infinite
2. The set {f(x) : x $\in \mathbb{R}$} is bounded
3. The set {f(x) : -1 < x < 1} is bounded
4. f(z) has infinitely many zeroes

JUNE – 2020

PART – B

88. Let γ be the positively oriented circle in the complex plane given by {z $\in \mathbb{C}$: |z – 1| = 1}.
Then $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z^3 - 1}$ equals

1. 3	2. 1/3
3. 2	4. ½

89. For a positive integer p, consider the holomorphic function
$f(z) = \frac{\sin z}{z^p}$ for z $\in \mathbb{C}\backslash\{0\}$.
For which values of p does there exist a holomorphic function g : $\mathbb{C}\backslash\{0\} \to \mathbb{C}$ such that f(z) = g′(z) for z $\in \mathbb{C}\backslash\{0\}$?
1. All even integers
2. All odd integers
3. All multiples of 3
4. All multiples of 4

90. Let γ be the positively oriented circle in the complex plane given by {z $\in \mathbb{C}$: |z – 1| = 1/2}. The line integral $\int_\gamma \frac{ze^{1/z}}{z^2 - 1} dz$ equals

1. $i\pi e$	2. $-i\pi e$
3. πe	4. $-\pi e$

91. Let p be a positive integer. Consider the closed curve $r(t) = e^{it}$, $0 \le t < 2\pi$. Let f be a function holomorphic in {z: |z| < R} where R > 1. If f has a zero only at z_0, $0 < |z_0| < R$, and it is of multiplicity q, then $\frac{1}{2\pi i}\int_r \frac{f'(z)}{f(z)} z^p dz$ equals

1. qz_0^p	2. $z_0 q^p$
3. pz_0^q	4. $z_0 p^q$

PART – C

92. For z ≠ -i, let $f(z) = \exp\left(\frac{1}{z+i}\right) - 1$. Which of the following are true?
1. f has finitely many zeroes
2. f has a sequence of zeroes that converges to a removable singularity of f
3. f has a sequence of zeroes that converges to a pole of f
4. f has a sequence of zeroes that converges to an essential singularity of f

93. Let f be a holomorphic function on the open unit disc

$\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. Suppose that $|f| \geq 1$ on $\mathbb{D}$ and $f(0) = i$.
Which of the following are possible values of $f\left(\frac{1}{2}\right)$?

1. $-i$ 2. i
3. 1 4. -1

94. Let $\mathbb{D} = \{z \in \mathbb{C}: |z| < 1\}$ be the open unit disc and let $f : \mathbb{D} \to \mathbb{D}$ be a holomorphic function. Suppose that $f(0) = 0$ and $f'(0) = 0$. Which of the following are possible values of $f\left(\frac{1}{2}\right)$?

1. 1/4 2. -1/4
3. 1/3 4. -1/3

95. Let n be a positive integer. For a real number
$R > 1$, let $z(\theta)= Re^{i\theta}$, $0 \leq \theta < 2\pi$.
The set
$\{ \theta \in [0, 2\pi) : |z(\theta)^n + 1| = |z(\theta)|^n - 1\}$ contains which of the following sets?
1. $\{ \theta \in [0, 2\pi): \cos n\theta = 1\}$
2. $\{ \theta \in [0, 2\pi): \sin n\theta = 1\}$
3. $\{ \theta \in [0, 2\pi): \cos n\theta = -1\}$
4. $\{ \theta \in [0, 2\pi): \sin n\theta = -1\}$

JUNE – 2021

PART – B

96. Let f(z) be a non-constant entire function and $z = x + iy$. Let $u(x, y)$, $v(x, y)$ denote its real and imaginary parts respectively. Which of the following statements is FALSE?
1. $u_x = v_y$ and $u_y = -v_x$
2. $u_y = v_x$ and $u_x = -v_y$
3. $|f'(x + iy)|^2 = u_x(x, y)^2 + v_x(x, y)^2$
4. $|f'(x + iy)|^2 = u_y(x, y)^2 + v_y(x, y)^2$

97. Let f be a rational function of a complex variable z given by $f(z) = \dfrac{z^3 + 2z - 4}{z}$.
The radius of convergence of the Taylor series of f at z = 1 is
1. 0 2. 1
3. 2 4. ∞

98. Let γ be the positively oriented circle $\{z\in\mathbb{C}: |z| = 3/2\}$.
Suppose that

$$\int_{\gamma} \frac{e^{i\pi z}}{(z-1)(z-2i)^2}\,dz = 2\pi i C.$$

Then |C| equals
1. 2 2. 5
3. 1/2 4. 1/5

99. Let $\mathbb{D}\subset\mathbb{C}$ be the open disc $\{z \in\mathbb{C}: |z| < 1\}$ and $O(\mathbb{D})$ be the space of all holomorphic functions on $\mathbb{D}$. Consider the sets

$$A = \left\{f \in O(\mathbb{D}): f\left(\frac{1}{n}\right) = \begin{cases} e^{-n} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases};\ for\ n \geq 2\right\}$$

$B = \{f \in O(\mathbb{D}): f(1/n) = (n - 2)/(n - 1), n \geq 2\}$.
Which of the following statements is true?
1. Both A and B are non-empty
2. A is empty and B has exactly one element
3. A has exactly one element and B is empty
4. Both A, B are empty

PART – C

100. For any complex valued function f, let D_f denote the set on which the function f satisfies Cauchy-Riemann equations. Identify the functions for which D_f is equal to $\mathbb{C}$.

(1) $f(z) = \dfrac{z}{1 + |z|}$

(2) $f(z) = (\cos \alpha x - \sin \alpha y) + i(\sin \alpha x + \cos \alpha y)$, where $z = x + iy$

(3) $f(z) = \begin{cases} e^{-\frac{1}{z^4}} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$

(4) $f(z) = x^2 + iy^2$, where $z = x + iy$

101. Let $\mathbb{T}$ denote the unit circle $\{z \in\mathbb{C}: |z| = 1\}$ in the complex plane and let $\mathbb{D}$ be the open unit disc $\{z \in\mathbb{C}: |z| < 1\}$. Let R denote the set of points z_0 in $\mathbb{T}$ for which there exists a holomorphic function f in an open neighbourhood U_{z_0} of z_0 such that

$f(z)=\sum_{n=0}^{\infty} z^{4n}$ in $U_{z_0} \cap \mathbb{D}$. Then R contains
(1) All points of $\mathbb{T}$
(2) Infinitely many points of $\mathbb{T}$
(3) All points of $\mathbb{T}$ except a finite set
(4) No points of $\mathbb{T}$

102. Consider the function
$$f(z)=\frac{(\sin z)^m}{(1-\cos z)^n} \; for\, 0<|z|<1 \text{ where}$$
m, n are positive integers. Then z = 0 is
(1) A removable singularity if m ≥ 2n
(2) A pole if m < 2n
(3) A pole if m ≥ 2n
(4) An essential singularity for some values of m, n

103. Let f be an entire function such that $|zf(z) - 1 + e^z| \le 1 + |z|$ for all $z \in \mathbb{C}$. Then
(1) f′(0) = -1
(2) f′(0) = -1/2
(3) f″(0) = -1/3
(4) f″(0) = -1/4

JUNE – 2022

PART – B

104. If $|e^{e^z}|=1$ for a complex number z=x + iy, x, y ∈ $\mathbb{R}$, then which of the following is true?
1. x = nπ for some integer n
2. $y=(2n+1)\frac{\pi}{2}$ for some integer n
3. y = nπ for some integer n
4. $x=(2n+1)\frac{\pi}{2}$ for some integer n

105. Let $f(z)=(1-z)e^{\left(z+\frac{z^2}{2}\right)}=1+\sum_{n=1}^{\infty} a_n z^n$. Which of the following is false?
1. $f'(z)=-z^2 e^{\left(z+\frac{z^2}{2}\right)}$
2. $a_1 = a_2$
3. $a_n \in (-\infty, 0]$
4. $\sum_{n=3}^{\infty} |a_n| < 1$

106. Let f be a non-constant entire function such that |f(z)| = 1 for |z| = 1. Let U denote the open unit disk around 0. Which of the followng is False?
1. $f(\mathbb{C}) = \mathbb{C}$
2. f has atleast one zero in U
3. f has atmost finitely many distinct zeroes in $\mathbb{C}$
4. f can have a zero outside U

107. For a positive integer n, let $f^{(n)}$ denote the n^{th} derivative of f. Suppose an entire function f satisfies $f^{(2)} + f = 0$. Which of the following is correct?
1. $(f^{(n)}(0))_{n\ge1}$ is convergent
2. $\lim_{n\to\infty} f^{(n)}(0) = 1$
3. $\lim_{n\to\infty} f^{(n)}(0) = -1$
4. $(|f^{(n)}(0)|)_{n\ge1}$ has a convergent subsequence

PART – C

108. For a bounded open connected subset Ω of $\mathbb{C}$, let f : Ω→ $\mathbb{C}$ be holomorphic. Let (z_k) be a sequence of distinct complex numbers in Ω converging to z_0. If $f(z_k) = 0$ for all k ≥ 1 then which of the following statements are necessarily true?
1. If f is non-zero, then $z_0 \in \partial\Omega$
2. There exists r > 0 such that f(z) = 0 for every z ∈ Ω satisfying $|z - z_0| \le r$
3. If $z_0 \in \Omega$, there exists r > 0 such that f(z) = 0 on $|z - z_0| = r$
4. $z_0 \in \partial\Omega$

109. Let f be an entire function such that $f(z)^2 + f'(z)^2 = 1$. Consider the following sets
X = {z : f′(z) = 0}, Y = {z : f″(z) + f(z) = 0}.
Which of the following statements are true?
1. Either X or Y has a limit point
2. If Y has a limit point, then f′ is constant
3. If X has a limit point, then f is constant
4. f(z) ∈ {1, -1} for all z ∈ $\mathbb{C}$

110. Let U be a bounded open set of $\mathbb{C}$ containing 0. Let f : U → U be holomorphic with f(0) = 0. For n ∈ $\mathbb{N}$, let f^n denote the composition of f done n times, that is, $f^n = \underbrace{f \circ \ldots \circ f}_{n\, times}$ while f′ denotes the derivative of f. Which of the following statements are true?
1. $(f^n)'(0) = (f'(0))^n$
2. $f^n(U) \subset U$
3. The sequence $((f'(0))^n)_n$ is bounded
4. |f′(0)| ≤ 1

111. For an open subset Ω of $\mathbb{C}$ such that $0 \in \Omega$, which of the following statements are true?

1. $\{e^z : z \in \Omega\}$ is an open subset of $\mathbb{C}$
2. $\{|e^z| : z \in \Omega\}$ is an open subset of $\mathbb{R}$
3. $\{\sin z: z \in \Omega\}$ is an open subset of $\mathbb{C}$
4. $\{|\sin z| : z \in \Omega\}$ is an open subset of $\mathbb{R}$

JUNE – 2023

PART – B

112. Let C be the positively oriented circle in the complex plane of radius 3 centered at the origin. What is the value of the integral

$$\int_C \frac{dz}{z^2(e^z - e^{-z})}?$$

(1) $i\pi/12$ (2) $-i\pi/12$
(3) $i\pi/6$ (4) $-i\pi/6$

113. Consider the function f defined by $f(z) = \dfrac{1}{1-z-z^2}$ for $z \in \mathbb{C}$ such that $1 - z - z^2 \neq 0$. Which of the following statements is true?

(1) f is an entire function
(2) f has a simple pole at z = 0
(3) f has a Taylor series expansion $f(z) = \sum_{n=0}^{\infty} a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0 = 1$, $a_1 = 0$ and $a_{n+2} = a_n + a_{n+1}$ for $n \geq 0$
(4) f has a Taylor series expansion $f(z) = \sum_{n=0}^{\infty} a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0 = 1$, $a_1 = 1$ and $a_{n+2} = a_n + a_{n+1}$ for $n \geq 0$

114. Let f be an entire function that satisfies $|f(z)| \leq e^y$ for all $z = x + iy \in \mathbb{C}$, where $x, y \in \mathbb{R}$. Which of the following statements is true?

(1) $f(z) = ce^{-iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(2) $f(z) = ce^{iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(3) $f(z) = e^{-ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(4) $f(z) = e^{ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$

115. Let $f(z) = \exp\left(z + \dfrac{1}{z}\right)$, $z \in \mathbb{C}\setminus\{0\}$. The residue of f at z = 0 is

(1) $\sum_{l=0}^{\infty} \dfrac{1}{(l+1)!}$
(2) $\sum_{l=0}^{\infty} \dfrac{1}{l!(l+1)}$
(3) $\sum_{l=0}^{\infty} \dfrac{1}{l!(l+1)!}$
(4) $\sum_{l=0}^{\infty} \dfrac{1}{(l^2+l)!}$

PART – C

116. Let f(z) be an entire function on $\mathbb{C}$. Which of the following statements are true?

(1) $f(\bar{z})$ is an entire function
(2) $\overline{f(z)}$ is an entire function
(3) $\overline{f(\bar{z})}$ is an entire function
(4) $\overline{f(z)} + f(\bar{z})$ is an entire function

117. Let $\mathbb{D} = \{z \in \mathbb{C}: |z| < 1\}$ be the open unit disc and C the positively oriented boundary $\{|z| = 1\}$. Fix a finite set $\{z_1, z_2, \dots, z_n\} \subseteq \mathbb{D}$ of distinct points and consider the polynomial
$g(z) = (z - z_1)(z - z_2) \dots (z - z_n)$
of degree n. Let f be a holomorphic function in an open neighbourhood of $\overline{\mathbb{D}}$ and define

$$P(z) = \frac{1}{2\pi i}\int_C f(\zeta)\frac{g(\zeta) - g(z)}{(\zeta - z)g(\zeta)}d\zeta.$$

Which of the following statements are true?

(1) P is a polynomial of degree n
(2) P is a polynomial of degree n – 1
(3) P is a rational function on $\mathbb{C}$ with poles at $z_1, z_2, \dots, z_n$
(4) $P(z_j) = f(z_j)$ for $j = 1, 2, \dots, n$.

118. Let $D = \{z \in \mathbb{C}: |z| < 1\}$. Consider the following statements.

(a) $f : D \to D$ be a holomorphic function. Suppose α, β are distinct complex numbers in D such that $f(\alpha) = \alpha$ and $f(\beta) = \beta$. Then $f(z) = z$ for all $z \in D$.
(b) There does not exist a bijective holomorphic function from D to the set of all complex numbers whose imaginary part is positive.

(c) $f : D \to D$ be a holomorphic function. Suppose $\alpha \in D$ be such that $f(\alpha) = \alpha$ and $f'(\alpha) = 1$. Then $f(z) = z$ for all $z \in D$.

Which of the following options are true?
(1) (a), (b) and (c) are all true.
(2) (a) is true.
(3) Both (a) and (b) are false.
(4) Both (a) and (c) are true.

119. Let $f : \{z : |z| < 1\} \to \{z : |z| \leq 1/2\}$ be a holomorphic function such that $f(0) = 0$. Which of the following statements are true?
(1) $|f(z)| \leq |z|$ for all z in $\{z : |z| < 1\}$.
(2) $|f(z)| \leq \left|\frac{z}{2}\right|$ for all z in $\{z : |z| < 1\}$
(3) $|f(z)| \leq 1/2$ for all z in $\{z : |z| < 1\}$
(4) It is possible that $f(1/2) = 1/2$.

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PART – B

120. Let $\mathbb{H} = \{z \in \mathbb{C}: \text{Im}(z) > 0\}$ denote the upper half plane and let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = e^{iz}$. Which one of the following statements is true?
(1) $f(\mathbb{H}) = \mathbb{C}\backslash\{0\}$.
(2) $f(\mathbb{H}) \cap \mathbb{H}$ is countable.
(3) $f(\mathbb{H})$ is bounded.
(4) $f(\mathbb{H})$ is a convex subset of $\mathbb{C}$.

121. How many roots does the polynomial $z^{100} - 50z^{30} + 40z^{10} + 6z + 1$ have in the open disc $\{z \in \mathbb{C} : |z| < 1\}$?
(1) 100 (2) 50
(3) 30 (4) 0

122. Let f be a meromorphic function on an open set containing the unit circle C and its interior. Suppose that's f has no zeros and no poles on C and let n_p and n_0 denote the number of poles and zeros of f inside C respectively. Which one of the following is true?
(1) $\frac{1}{2\pi i}\int_C \frac{(zf)'}{zf}dz = n_0 - n_p + 1.$
(2) $\frac{1}{2\pi i}\int_C \frac{(zf)'}{zf}dz = n_0 - n_p - 1.$
(3) $\frac{1}{2\pi i}\int_C \frac{(zf)'}{zf}dz = n_0 - n_p.$
(4) $\frac{1}{2\pi i}\int_C \frac{(zf)'}{zf}dz = n_p - n_0.$

123. Let $f : \mathbb{C} \to \mathbb{C}$ be a real-differentiable function. Define $u, v : \mathbb{R}^2 \to \mathbb{R}$ by $u(x, y) = \text{Re } f(x + iy)$ and $v(x, y) = \text{Im } f(x + iy)$, $x, y \in \mathbb{R}$.

Let $\nabla u = (u_x, u_y)$ denote the gradient. Which one of the following is necessarily true?
(1) For $c_1, c_2 \in \mathbb{C}$, the level curves $u = c_1$ and $v = c_2$ are orthogonal wherever they intersect.
(2) $\nabla u \cdot \nabla v = 0$ at every point.
(3) If f is an entire function, then $\nabla u \cdot \nabla v = 0$ at every point.
(4) If $\nabla u \cdot \nabla v = 0$ at ever point, then f is an entire function.

PART – C

124. Let $\Omega_1 = \{z \in \mathbb{C} : |z| < 1\}$ and $\Omega_2 = \mathbb{C}$. Which of the following statements are true?
(1) There exists a holomorphic surjective map $f : \Omega_1 \to \Omega_2$.
(2) There exists a holomorphic surjective map $f : \Omega_2 \to \Omega_1$.
(3) There exists a holomorphic injective map $f : \Omega_1 \to \Omega_2$.
(4) There exists a holomorphic injective map $f : \Omega_2 \to \Omega_1$.

125. For every $n \geq 1$, consider the entire function $p_n(z) = \sum_{k=0}^{n} \frac{z^k}{k!}$. Which of the following statements are true?
(1) The sequence of functions $(p_n)_{n\geq 1}$ converges to an entire function uniformly on compact subsets of $\mathbb{C}$.
(2) For all $n \geq 1$, p_n has a zero in the set $\{z \in \mathbb{C} : |z| \leq 2023\}$.
(3) There exists a sequence (z_n) of complex numbers such that $\lim_{n\to\infty} |z_n| = \infty$ and $p_n(z_n) = 0$ for all $n \geq 1$.
(4) Let S_n denote the set of all the zeros of p_n. If $a_n = \min_{z \in S_n} |z|$, then $a_n \to \infty$ as $n \to \infty$.

126. Let X be an uncountable subset of $\mathbb{C}$ and let $f : \mathbb{C} \to \mathbb{C}$ be an entire function. Assume that for every $z \in X$, there exists an integer $n \geq 1$ such that $f^{(n)}(z) = 0$. Which of the following statements are necessarily true?
(1) f = 0.
(2) f is a constant function.
(3) There exists a compact subset K of $\mathbb{C}$ such that $f^{-1}(K)$ is not compact.
(4) f is a polynomial.

127. For an integer k, consider the contour integral $I_k = \int_{|z|=1} \frac{e^z}{z^k} dz$. Which of the following statements are true?
(1) $I_k = 0$ for every integer k.
(2) $I_k \neq 0$ if $k \geq 1$.
(3) $|I_k| \leq |I_{k+1}|$ for every integer k.
(4) $\lim_{k\to\infty} |I_k| = \infty$.

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PART – B

128. Consider the contour γ given by

$$\gamma(\theta) = \begin{cases} e^{2i\theta} & for\ \theta \in [0, \pi/2] \\ 1+2e^{2i\theta} & for\ \theta \in [\pi/2, 3\pi/2] \\ e^{2i\theta} & for\ \theta \in [3\pi/2, 2\pi] \end{cases}$$

Then what is the value of $\int_\gamma \frac{dz}{z(z-2)}$?
(1) 0 (2) πi
(3) $-\pi i$ (4) $2\pi i$

129. Let f be an entire function. Which of the following statements is FALSE?
(1) If Re(f), Im(f) are bounded then f is constant
(2) If $e^{|Re(f)| + |Im(f)|}$ is bounded, then f is constant
(3) If the sum Re(f) + Im(f) and the product Re(f) Im(f) are bounded, the f is constant
(4) If sin (Re(f) + Im(f)) is bounded, then f is constant

130. Let a, b be two real numbers such that a < 0 < b. For a positive real number r, define $\gamma_r(t) = re^{it}$ (where $t \in [0, 2\pi]$) and $I_r = \frac{1}{2\pi i}\int_{\gamma r} \frac{z^2+1}{(z-a)(z-b)} dz$. Which of the followig statements is necessarily true?
(1) $I_r \neq 0$ if r > max {|a|, b}
(2) $I_r \neq 0$ if r < max {|a|, b}
(3) $I_r = 0$ if r > max {|a|, b} and |a| = b
(4) $I_r = 0$ if |a| < r < b

131. For a complex number a such that 0 < |a| < 1, which of the following statements is true?
(1) $If\ |z|<1, then |1-\bar{a}z| < |z-a|$
(2) $If\ |z-a| = |1-\bar{a}z|, then |z| = 1$
(3) $If\ |z| = 1, then |z-a| < |1-\bar{a}z|$
(4) $If\ |1-\bar{a}z| < |z-a|, then |z| < 1$

PART – C

132. Which of the following conditions ensure that the power series $\sum_{n\geq 0} a_n z^n$ defines an entire function?
(1) The power series converges for every $z \in \mathbb{C}$
(2) The power series converges for every $z \in \mathbb{R}$
(3) The power series converges for every $z \in \{2^n : n \in \mathbb{N}\}$
(4) The power series converges for every $z \in \left\{\frac{1}{5^n} : n \in \mathbb{N}\right\}$

133. Let f be an entire function such that for every integer $k \geq 1$ there is an infinite set X_k such that $f(z) = \frac{1}{k}$ for all $z \in X_k$. Which of the following statements are necessarily true?
(1) There exists an infinite set X such that f(z) = 0 for all $z \in X$
(2) There exists a non-empty closed set X such that f(z) = 0 for all $z \in X$
(3) The set X_k is unbounded for each $k \geq 1$
(4) If there exists a bounded sequence $(z_k)_{k\geq 1}$ such that $z_k \in X_k$ for each $k \geq 1$, then f has a zero

134. Suppose that f is an entire function such that $|f(z)| \geq 2024$ for all $z \in \mathbb{C}$. Which of the following statements are necessarily true?
(1) f(z) = 2024 for all $z \in \mathbb{C}$
(2) f is a constant function
(3) f is an injective function

(4) f is a bijective function

135. For $z \in \mathbb{C}\setminus\{0\}$, let $f(z) = \frac{1}{z}\sin\left(\frac{1}{z}\right)$ and $g(z) = f(z)\sin(z)$. Which of the following statements are true?
(1) f has an essential singularity at 0
(2) g has an essential singularity at 0
(3) f has a removable singularity at 0
(4) g has a removable singularity at 0

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PART – B

136. Let f: $\mathbb{C} \to \mathbb{C}$ be the function defined by $f(z) = e^{(\cos(1+i)\sin z)}$. For $z = x + iy \in \mathbb{C}$, write f(z) as $u(x,y) + i\upsilon(x,y)$, where u, v are real-valued functions . Which of the following is the value of $\frac{\partial u}{\partial x}(0,0)$?
(1)0
(2) $\left(e+\frac{1}{e}\right)\frac{\cos 1}{2}$
(3) $\left(e-\frac{1}{e}\right)\frac{\cos 1}{2}$
(4)1

137. Let $\mathbb{D} = \{z = x + iy \in \mathbb{C} : |z| < 1\}$ be the open unit disc and $f : \mathbb{D} \to \mathbb{C}$ a holomorphic function such that $f(0) = 0. Let\, \psi(z) = |f(z)|^2,\ and$
$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0.$$
Which of the following statements is FALSE?
(1) f can be extended to $\mathbb{C}$ as an entire function.
(2) f must have infinitely many zeros in $\mathbb{D}$
(3) f is not a polynomial.
(4) exp(f) cannot take every complex value.

138. For integers $m, n \geq 1, let\ I_{m,n} = \frac{1}{2\pi i}\int_C z^m \bar{z}^n dz$, where C is the circle $\{z \in \mathbb{C} : |z| = 1\}$ oriented counter clockwise. Which of the following statements is true?
(1) $I_{m,n} = 1\ if\ m = n$
(2) $I_{m,n} = 1\ if\ m + 1 = n$
(3) $I_{m,n} = 1\ if\ m = n + 1$
(4) $I_{m,n} = 1\ if\ m = n + 2$

139. Let $\mathbb{H} = \{z = x + iy \in \mathbb{C} \mid y > 0\}\ and\ f : \mathbb{H} \to \mathbb{C}$ be a non-constant holomorphic function satisfying $|f(z)| < 1\ for\ all\ z \in \mathbb{H}$. Which of the following statements is true?
(1) $\lim_{y\to+\infty} f'(iy) = 0$
(2) $\lim_{y\to+\infty} f'(iy)$ is a complex number with absolute value 1.
(3) $\lim_{y\to+\infty} |f'(iy)| = +\infty$
(4) $\lim_{y\to+\infty} f'(iy)$ is not a real number.

PART – C

140. Let f: $\mathbb{C} \to \mathbb{C}$ be an entire function such that f(z) = f(iz) for all $z \in \mathbb{C}$. Which of the following statements are true?
(1) $f(z) = f(-z)\ for\ all\ z \in C.$
(2) $f'(0) = f''(0) = f'''(0) = 0$
(3) There is an entire function g: $\mathbb{C} \to \mathbb{C}$ such that $f(z) = g(z^4)$ for all $z \in \mathbb{C}$.
(4) f is necessarily a constant function.

141. Let p(z) be a non-constant polynomial over $\mathbb{C}$. Given R>0, let $S_R = \{z \in \mathbb{C} : |P(z)| < R\}$ Which of the following statements are true?
(1) S_R is an open subset of $\mathbb{C}$.
(2) S_R is a bounded subset of $\mathbb{C}$.
(3) |P(z)|=R for every z on the boundary of S_R.
(4) Every connected component of S_R contains a zero of p(z).

142. Let disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and f be a holomorphic function on $\mathbb{D}$ such that the function g(z)=$e^{1/z}$ f(z) on $\mathbb{D} \setminus \{0\}$ is bounded
. Which of the following statements are true?
(1) f(0)=0
(2) f(z)=0 for all $z \in \mathbb{D}$
(3) There exists a nonzero constant c such that f(z)= c$e^{-1/z}$ for all $z \in \mathbb{D}\setminus\{0\}$.

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(4)There exists a nonzero constant c and a positive integer n such that $f(z) = cz^n e^{-1/z}$ *for all* $z \in \mathbb{D} \setminus \{0\}$.

143. Let f: $\mathbb{C} \setminus \{-1,1\} \to \mathbb{C}$ be a holomorphic function that does not take any value in the set $\{z \in \mathbb{C} : |z-1| < 1\}$. Which of the following statements are true?
(1) f is constant.
(2) f has removable singularities at -1 and 1.
(3) f is bounded.
(4) f has either poles or essential singularities at -1 and 1.

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PART – B

144. Let f be an entire function such that f(ℂ) ⊂ {x + iy | y = x + 1}. Which of the following statements is true?
(1) |f(z)| →∞ as |z| →∞.
(2) $\frac{f(z)}{z} \to 0$ as |z| →∞
(3) zf(z) → 0 as |z| →∞
(4) f(z) → 0 as |z| →∞

145. Which of the following statements is true?
(1) There exists an entire function f such that $f^{(n)}(0) = \frac{n!}{n^n}$ for all positive integers n.
(2) There exists an entire function f such that $f^{(n)}(0) = n!n^n$ for all positive integers n.
(3) There exists an entire function f such that $f^{(n)}(0) = (n-1)!$ for all positive integers n.
(4) There exists an entire function f such that $f^{(n)}(0) = n!n$ for all positive integers n.

146. Let f : ℂ→ℂ be a polynomial map. For R > 0, let γ_R : [0, 1] →ℂ be the map $t \mapsto Re^{2\pi it}$. Suppose that there exists c ∈ℝ such that
$$\int_0^1 |(f \circ \gamma_R)(t)\gamma_R'(t)|dt \to c \text{ as } R \to \infty.$$
Which of the following statements if False?
(1) The function zf (1/z) → 0 as |z| →∞.
(2) The function f is constant.
(3) c = 0
(4) c > 0

147. Let X be the image of the interval [0, 1] under the *Möbius* transformation $f(z) = \frac{z-i}{z+i}$. Which of the following statements is true?
(1) X is the line segment joining -1 and –i.
(2) $X = \left\{ e^{i\theta} \mid \theta \in \left[\pi, \frac{3\pi}{2}\right] \right\}$.
(3) X is the line segment joining -1 to 1.
(4) $X = \left\{ e^{i\theta} \mid \theta \in \left[-\frac{\pi}{2}, \pi\right] \right\}$.

PART – C

148. Let $D^\times = \{z \in \mathbb{C} : 0 < |z| < 1\}$ be the punctured unit disk and f be a bijective holomorphic map of $D^\times$ onto itself. Which of the following statements are true?
1. $\lim_{z\to 0} f(z)$ does not exist.
2. $\lim_{z\to 0} f(z)$ exists and has absolute value ≤ 1.
3. $\lim_{z\to 0} f(z) = 0$
4. There exists $\theta \in \mathbb{R}$ such that $f(z) = e^{i\theta}z$ for all $z \in D^\times$

149. Let f be an entire function. Which of the following are true?
1. If f(z)= f(z+1) for all z ∈ ℂ then f is a constant function.
2. If f(z) = f(z+1) = f(z+i) for all z ∈ ℂ then f is a constant function
3. If $f\left(\frac{1}{z}\right)$ has a removable singularity at 0 then f is a constant function
4. If f is a non-constant function then $f\left(\frac{1}{z}\right)$ has a pole at 0.

150. Let f be an entire function which is not a polynomial. Let $A = \{\alpha \in \mathbb{C} \mid f^{(n)}(\alpha) \neq 0 \text{ for all } n \geq 0\}$.
Which of the following statements are true?
1. A is nonempty

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2. A is finite
3. A is infinite
4. A is uncountable.

151. Let $\gamma:[0,1]\to\mathbb{C}$ be the function $t\to e^{2\pi it}$

and $I=\int_{\gamma} e^{z}e^{\frac{1}{z}}dz.$

Which of the following statements are true?

1. I = 0
2. $\frac{1}{2\pi i}I\in\{4n:n\in,\mathbb{Z}\ \ n\geq 1\}$
3. $I=2\pi i\sum_{n=0}^{\infty}\frac{1}{n!}$
4. $I=2\pi i\sum_{n=0}^{\infty}\frac{1}{n!(n+1)!}$

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ANSWERS

1. (2)	2. (1)	3. (1,4)
4. (3)	5. (2,3,4)	6. (1,2)
7. (1)	8. (1)	9.
10. (2)	11. (1,2,3)	12. (1,3)
13. (3)	14. (3)	15. (2)
16. (1,2,4)	17. (1,2,3,4)	18. (1,2,3)
19. (1)	20. (3)	21. (2)
22. (1,2,3,4)	23. (1,2,3,4)	24. (2,3)
25. (3)	26. (2)	27. (2)
28. (3)	29. (1,2,4)	30. (3,4)
31. (1,2,3,4)	32. (1,2,4)	33. (3)
34. (3)	35. (4)	36. (2)
37. (2,3)	38. (2,4)	39. (1,2)
40. (1,2,4)	41. (3)	42. (2)
43. (1)	44. (3)	45. (1,2)
46. (3,4)	47. (2,4)	48. (2,4)
49. (4)	50. (3)	51. (4)
52. (3)	53. (1,3)	54. (1,2,3,4)
55. (1,2,3)	56. (1,3)	57. (2)
58. (1)	59. (4)	60. (3)
61. (1,3)	62. (1,2,3,4)	63. (1,2,4)
64. (1,3)	65. (3)	66. (1)
67. (2)	68. (4)	69. (1,2)
70. (1,3)	71. (2,4)	72. (1)
73. (1)	74. (2)	75. (3)
76. (1,4)	77. (1,2,3,4)	78. (2,4)
79. (1,3)	80. (4)	81. (3)
82. (3)	83. (3)	84. (1)
85. (1,2,4)	86. (1,2,3,4)	87. (1,3,4)
88. (2)	89. (2)	90. (1)
91. (1)	92. (4)	93. (2)
94. (1,2)	95. (3)	96. (2)
97. (2)	98. (4)	99. (2)
100. (3)	101. (2,3)	102. (1,2)
103. (2,3)	104. (2)	105. (4)
106. (4)	107. (4)	108. (1,3)
109. (1,3)	110. (1,2,3,4)	111.(1,2,3)
112. (4)	113. (4)	114. (1)
115. (3)	116. (3)	117. (4)
118. (2,4)	119. (1,2,3)	120. (3)
121. (3)	122. (1)	123. (3)
124. (1,3)	125. (1,3,4)	126. (4)
127. (2)	128. (3)	129. (4)
130. (3)	131. (2)	132. (1,2,3)
133. (3,4)	134. (2)	135. (1,2)
136. (2)	137. (3)	138. (2)
139. (1)	140. (1,2,3)	141. (1,2,3,4)
142. (1,2)	143. (1,2,3)	144. (2)
145. (1)	146. (4)	147. (2)
148. (2,3,4)	149. (2,3)	150. (1,3,4)
151. (4)		

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