



INTEGRAL EQUATION ASSIGNMENT

JUNE - 2014

PART - B

1. The homogeneous integral equation

$$\varphi(x) - \lambda \int_0^1 (3x-2)t \varphi(t) dt = 0, \text{ has}$$

1. One characteristic number
2. Three characteristic numbers
3. Two characteristic numbers
4. No characteristic number

PART - C

2. Let λ_1, λ_2 be the characteristic numbers and f_1, f_2 the corresponding eigen functions for the homogeneous integral equation

$$\varphi(x) - \lambda \int_0^1 (xt + 2x^2) \varphi(t) dt = 0. \text{ Then}$$

1. $\lambda_1 = -18 - 6\sqrt{10}, \lambda_2 = -18 + 6\sqrt{10}$
2. $\lambda_1 = -36 - 12\sqrt{10}, \lambda_2 = -36 + 12\sqrt{10}$
3. $\int_0^1 f_1(x) f_2(x) dx = 1$
4. $\int_0^1 f_1(x) f_2(x) dx = 0$

3. If $y: [0, \infty) \rightarrow [0, \infty)$ is a continuously differentiable function satisfying

$$y(t) = y(0) - \int_0^t y(s) ds \text{ for } t \geq 0, \text{ then}$$

1. $y^2(t) = y^2(0) + \left(\int_0^t y(s) ds \right)^2 - 2y(0) \int_0^t y(s) ds$
2. $y^2(t) = y^2(0) + 2 \int_0^t y^2(s) ds$
3. $y^2(t) = y^2(0) - \int_0^t y(s) ds$
4. $y^2(t) = y^2(0) - 2 \int_0^t y^2(s) ds$

DEC - 2014

4. Let $y: [0, \infty) \rightarrow \mathbb{R}$ be twice continuously differentiable and satisfy

$$y(x) + \int_0^x (x-s)y(s) ds = x^3/6. \text{ Then}$$

1. $y(x) = \frac{1}{6} \int_0^x s^3 \sin(x-s) ds$
2. $y(x) = \frac{1}{6} \int_0^x s^3 \cos(x-s) ds$

$$3. y(x) = \int_0^x s \sin(x-s) ds$$

$$4. y(x) = \int_0^x s \cos(x-s) ds$$

5. Let $u \in C^2([0,1])$ satisfy for some $\lambda \neq 0$ and $a \neq 0$

$$u(x) + \frac{\lambda}{2} \int_0^1 |x-s| u(s) ds = ax + b.$$

Then u also satisfies

1. $\frac{d^2 u}{dx^2} + \lambda u = 0$
2. $\frac{d^2 u}{dx^2} - \lambda u = 0$
3. $\frac{du}{dx} - \frac{\lambda}{2} \int_0^1 \frac{x-s}{|x-s|} u(s) ds = a$
4. $\frac{du}{dx} + \frac{\lambda}{2} \int_0^1 \frac{x-s}{|x-s|} u(s) ds = a$

JUNE-2015

6. The integral equation

$$y(x) = \lambda \int_0^1 (3x-2)ty(t) dt, \text{ with } \lambda \text{ as a parameter, has}$$

1. only one characteristic number
 2. two characteristic numbers
 3. more than two characteristic numbers
 4. no characteristic number
7. For the integral equation

$$y(x) = 1 + x^3 + \int_0^x K(x,t)y(t) dt \text{ with kernel}$$

$K(x,t) = 2^{x-t}$, the iterated kernel $K_3(x,t)$ is

1. $2^{x-t}(x-t)^2$
2. $2^{x-t}(x-t)^3$
3. $2^{x-t-1}(x-t)^2$
4. $2^{x-t-1}(x-t)^3$

DEC-2015

8. The resolvent kernel $R(x, t, \lambda)$ for the Volterra integral equation $\varphi(x) = x + \lambda \int_a^x \varphi(s) ds$, is

1. $e^{\lambda(x+t)}$
2. $e^{\lambda(x-t)}$
3. $\lambda e^{(x+t)}$
4. $e^{\lambda xt}$

9. Let $y: [0, \infty) \rightarrow [0, \infty)$ be a continuously differentiable function satisfying



$y(t) = y(0) + \int_0^t y(s) ds$ for $t \geq 0$. Then

1. $y^2(t) = y^2(0) + \int_0^t y^2(s) ds.$
2. $y^2(t) = y^2(0) + 2 \int_0^t y^2(s) ds.$
3. $y^2(t) = y^2(0) + \int_0^t y(s) ds.$
4. $y^2(t) = y^2(0) + \left(\int_0^t y(s) ds \right)^2 + 2y(0) \int_0^t y(s) ds.$

10. Let λ_1, λ_2 be the characteristic numbers and f_1, f_2 be the corresponding eigen functions for the homogenous integral equation

$$\phi(x) - \lambda \int_0^1 (2xt + 4x^2) \phi(t) dt = 0. \text{ Then}$$

1. $\lambda_1 \neq \lambda_2$
2. $\lambda_1 = \lambda_2$
3. $\int_0^1 f_1(x) f_2(x) dx = 0$
4. $\int_0^1 f_1(x) f_2(x) dx = 1$

JUNE – 2016

PART – B

11. Consider the integral equation

$$y(x) = x^3 + \int_0^x \sin(x-t) y(t) dt, x \in [0, \pi].$$

Then the value of $y(1)$ is

1. 19/20
2. 1
3. 17/20
4. 21/20

PART- C

12. The curve $y = y(x)$, passing through the point $(\sqrt{3}, 1)$ and defined by the following property (Volterra integral equation of the first kind)

$$\int_0^y \frac{f(v) dv}{\sqrt{y-v}} = 4\sqrt{y}, \text{ where } f(y) = \sqrt{1 + \frac{1}{y'^2}}, \text{ is}$$

the part of a

1. Straight line.
2. Circle
3. Parabola.
4. Cycloid.

DEC-2016

13. Let ϕ satisfy $\phi(x) = f(x) + \int_0^x \sin(x-t) \phi(t) dt.$

Then ϕ is given by

1. $\phi(x) = f(x) + \int_0^x (x-t) f(t) dt$

$$2. \phi(x) = f(x) - \int_0^x (x-t) f(t) dt$$

$$3. \phi(x) = f(x) - \int_0^x \cos(x-t) f(t) dt$$

$$4. \phi(x) = f(x) - \int_0^x \sin(x-t) f(t) dt$$

14. Which of the following are the characteristic numbers and the corresponding eigen-functions for the Fredholm homogeneous equation whose kernel is

$$K(x, t) = \begin{cases} (x+1)t, & 0 \leq x \leq t \\ (t+1)x, & t \leq x \leq 1 \end{cases} ?$$

1. 1, e^x
2. $-\pi^2, \pi \sin \pi x + \cos \pi x$
3. $-4\pi^2, \pi \sin \pi x + \pi \cos 2\pi x$
4. $-\pi^2, \pi \cos \pi x + \sin \pi x$

15. The integral equation

$$\phi(x) - \frac{2}{\pi} \int_0^\pi \cos(x+t) \phi(t) dt = f(x)$$

has infinitely many solutions if

1. $f(x) = \cos x$
2. $f(x) = \cos 3x$
3. $f(x) = \sin x$
4. $f(x) = \sin 3x$

JUNE – 2017

16. Let $\phi(x)$ be the solution of

$$\int_0^x e^{x-t} \phi(t) dt = x, \quad x > 0.$$

Then $\phi(1)$ equals

1. -1
2. 0
3. 1
4. 2

17. Let $y(x)$ be the solution of the integral

$$\text{equation } y(x) = x - \int_0^x t^2 y(t) dt, x > 0.$$

Then the value of the function $y(x)$ at $x = \sqrt{2}$ is equal to

1. $\frac{1}{\sqrt{2}e}$
2. $\frac{e}{2}$
3. $\frac{\sqrt{2}}{e^2}$
4. $\frac{\sqrt{2}}{e}$

18. The solutions for $\lambda = -1$ and $\lambda = 3$ of the integral equation

$$y(x) = 1 + \lambda \int_0^1 K(x, t) y(t) dt, \text{ where}$$



$K(x,t) = \begin{cases} \cosh x \sinh t, & 0 \leq x \leq t \\ \cosh t \sinh x, & t \leq x \leq 1 \end{cases}$ are, respectively,

1. $-\frac{x^2}{2} + \frac{3}{2} - \tanh 1$ and $\frac{1}{4} \left(\frac{3 \cos 2x}{\cos 2 - 2 \sin 2 \tanh 1} + 1 \right)$
2. $-\frac{x^2}{2} + \frac{3}{2} - \tanh 1$ and $\frac{1}{4} \left(\frac{3 \cos 2x}{\cosh 2 - 2 \sinh 2 \tanh 1} + 1 \right)$
3. $\frac{x^2}{2} + \frac{3}{2} - \tanh 1$ and $\frac{1}{4} \left(\frac{3 \cos 2x}{\cosh 2 - 2 \sinh 2 \tanh 1} - 1 \right)$
4. $\frac{x^2}{2} + \frac{3}{2} - \tanh 1$ and $\frac{1}{4} \left(\frac{3 \cos 2x}{\cos 2 - 2 \sin 2 \tanh 1} - 1 \right)$

JUNE - 2018

PART - B

19. The resolvent kernel for the integral equation $\phi(x) = x^2 + \int_0^x e^{t-x} \phi(t) dt$ is

1. e^{t-x}
2. 1
3. e^{x-t}
4. $x^2 + e^{x-t}$

PART - C

20. The values of λ for which the following equation has a non-trivial solution

$$\phi(x) = \lambda \int_0^\pi K(x,t) \phi(t) dt, 0 \leq x \leq \pi,$$

where $K(x,t) = \begin{cases} \sin x \cos t, & 0 \leq x \leq t \\ \cos x \sin t, & t \leq x \leq \pi \end{cases}$ are

1. $\left(n + \frac{1}{2}\right)^2 - 1, n \in \mathbb{N}$
2. $n^2 - 1, n \in \mathbb{N}$
3. $\frac{1}{2}(n+1)^2 - 1, n \in \mathbb{N}$
4. $\frac{1}{2}(2n+1)^2 - 1, n \in \mathbb{N}$

21. Consider the integral equation

$$\phi(x) = \lambda \int_0^\pi [\cos x \cos t - 2 \sin x \sin t] \phi(t) dt + \cos 7x, 0 \leq x \leq \pi$$

Which of the following statements are true?

1. For every $\lambda \in \mathbb{R}$, a solution exists
2. There exists $\lambda \in \mathbb{R}$ such that solution does not exist
3. There exists $\lambda \in \mathbb{R}$ such that there are more than one but finitely many solutions
4. There exists $\lambda \in \mathbb{R}$ such that there are infinitely many solutions

DECEMBER - 2018

PART - B

22. If ϕ is the solution of

$$\int_0^x (1 - x^2 + t^2) \phi(t) dt = \frac{x^2}{2}, \text{ then } \phi(\sqrt{2}) \text{ is}$$

equal to

1. $\sqrt{2}e^{\sqrt{2}}$
2. $\sqrt{2}e^2$
3. $\sqrt{2}e^{2\sqrt{2}}$
4. $2e^4$

PART-C

23. If ϕ is the solution of $\phi(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x-t) - 4(x-t)^2] \phi(t) dt$, then $\phi(\log 2)$ is equal to

1. 2
2. 4
3. 6
4. 8

24. A characteristic number and the corresponding eigenfunction of the homogenous Fredholm integral equation

$$\text{with kernel } K(x,t) = \begin{cases} x(t-1), & 0 \leq x \leq t \\ t(x-1), & t \leq x \leq 1 \end{cases}$$

are

1. $\lambda = -\pi^2, \phi(x) = \sin \pi x$
2. $\lambda = -2\pi^2, \phi(x) = \sin 2\pi x$
3. $\lambda = -3\pi^2, \phi(x) = \sin 3\pi x$
4. $\lambda = -4\pi^2, \phi(x) = \sin 2\pi x$



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PART – B

25. If y is a solution of $y(x) - \int_0^x (x-t)y(t)dt = 1$, then which of the following is true?
1. y is bounded but not periodic in $\mathbb{R}$
 2. y is periodic in $\mathbb{R}$
 3. $\int_{\mathbb{R}} y(x)dx < \infty$
 4. $\int_{\mathbb{R}} \frac{dx}{y(x)} < \infty$

PART – C

26. Consider the integral equation $\phi(x) - \frac{e}{2} \int_{-1}^1 x e^t \phi(t) dt = f(x)$. Then
1. there exists a continuous function $f : [-1, 1] \rightarrow (0, \infty)$ for which solution exists
 2. there exists a continuous function $f : [-1, 1] \rightarrow (-\infty, 0)$ for which solution exists
 3. for $f(x) = e^{-x}(1 - 3x^2)$, a solution exists
 4. for $f(x) = e^{-x}(x + x^3 + x^5)$, a solution exists

DECEMBER – 2019

PART - B

27. Let ϕ be the solution of $\phi(x) = 1 - 2x - 4x^2$
- $$+ \int_0^x [3 + 6(x-t) - 4(x-t)^2] \phi(t) dt.$$
- Then $\phi(1)$ is equal to
1. e^{-1}
 2. e^{-2}
 3. e
 4. e^2

PART – C

28. Assume that h_1, h_2, g_1 and $g_2 \in C([a, b])$. Let $\phi(x) = f(x)$
- $$+ \lambda \int_a^b [h_1(t)g_1(x) + h_2(t)g_2(x)] \phi(t) dt$$
- be an integral domain. Consider the following statements:

S_1 : If the given interval equation has a solution for some $f \in C([a, b])$, then

$$\int_a^b f(t)g_1(t)dt = 0 = \int_a^b f(t)g_2(t)dt.$$

S_2 : The given integral equation has a unique solution for every $f \in C([a, b])$ if λ is not a characteristic number of the corresponding homogeneous equation.

Then

1. Both S_1 and S_2 are true
2. S_1 is true but S_2 is false
3. S_1 is false but S_2 is true
4. Both S_1 and S_2 are false

29. The integral equation

$$\phi(x) = 1 + \frac{2}{\pi} \int_0^\pi (\cos^2 x) \phi(t) dt$$

has

1. no solution
2. unique solution
3. more than one but finitely many solutions
4. infinitely many solutions

JUNE – 20

PART – B

30. The solution of the Fredholm integral equation

$$y(s) = s + 2 \int_0^1 (st^2 + s^2t)y(t)dt \text{ is}$$

1. $y(s) = -(50s + 40s^2)$
2. $y(s) = (30s + 15s^2)$
3. $y(s) = -(30s + 40s^2)$
4. $y(s) = (60s + 50s^2)$

PART – C

31. For the Fredholm integral equation

$$y(s) = \lambda \int_0^1 e^s e^t y(t) dt$$

Which of the following statements are true?

1. It has a non-trivial solution satisfying $\int_0^1 e^t y(t) dt = 0$
2. Only the trivial solution satisfies $\int_0^1 e^t y(t) dt = 0$
3. It has non-trivial solution for all $\lambda \neq 0$
4. It has non-trivial solutions only if $\lambda = \frac{2}{e^2 - 1}$ and $\int_0^1 e^t y(t) dt \neq 0$



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PART – B

32. Consider the integral equation $\int_0^x (x-t)u(t)dt = x; x \geq 0$ for continuous functions u defined on $[0, \infty)$. The equation has

1. A unique bounded solution
2. No solution
3. More than one solution u such that $|u(x)| \leq C(1 + |x|)$ for some constant C
4. A unique solution u such that $|u(x)| \leq C(1 + |x|)$ for some constant C

PART – C

33. Let $K(x, y)$ be a kernel in $[0, 1] \times [0, 1]$, defined as $K(x, y) = \sin(2\pi x) \sin(2\pi y)$. Consider the integral operator

$$K(u)(x) = \int_0^1 u(y) K(x, y) dy \text{ where } u \in C([0, 1]).$$

Which of the following assertions on K are true?

1. The null space of K is infinite dimensional
2. $\int_0^1 v(x) K(u)(x) dx = \int_0^1 K(v)(x) u(x) dx$ for all $u, v \in C([0, 1])$
3. K has no negative eigenvalue
4. K has an eigenvalue greater than $3/4$

JUNE – 22

PART – B

34. For any two continuous functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ define

$$f * g(t) = \int_0^t f(s) g(t-s) ds.$$

Which of the following is the value of $f * g(t)$ when $f(t) = \exp(-t)$ and $g(t) = \sin(t)$.

1. $\frac{1}{2} [\exp(-t) + \sin(t) - \cos(t)]$
2. $\frac{1}{2} [-\exp(-t) + \sin(t) - \cos(t)]$
3. $\frac{1}{2} [\exp(-t) - \sin(t) - \cos(t)]$
4. $\frac{1}{2} [\exp(-t) + \sin(t) + \cos(t)]$

PART – C

35. Let g be the solution of the Volterra type integral equation

$$g(s) = 1 + \int_0^s (s-t) g(t) dt; \text{ for all } s \geq 0.$$

What are the possible values of $g(1)$?

1. $2e$
2. $e - \frac{1}{e}$
3. $e + \frac{1}{e}$
4. $\frac{2}{e}$

36. Consider the following system of Integral

$$\varphi_1(x) = \sin x + \int_0^x \varphi_2(t) dt$$

$$\varphi_2(x) = 1 - \cos x - \int_0^x \varphi_1(t) dt$$

Which of the following statements are true

1. φ_1 vanishes at most countably many points
2. φ_1 vanishes at uncountably many points
3. φ_2 vanishes at most countably many points
4. φ_2 vanishes at uncountably many points

JUNE – 23

PART – B

37. For the unknown $y : [0, 1] \rightarrow \mathbb{R}$, consider the following two-point boundary value problem:

$$\begin{cases} y''(x) + 2y(x) = 0 & \text{for } x \in (0, 1), \\ y(0) = y(1) = 0. \end{cases}$$

It is given that the above boundary value problem corresponds to the following integral equation:

$$y(x) = 2 \int_0^1 K(x, t) y(t) dt \quad \text{for } x \in [0, 1].$$

Which of the following is the kernel $K(x, t)$?

1. $K(x, t) = \begin{cases} t(1-x) & \text{for } t < x \\ x(1-t) & \text{for } t > x \end{cases}$
2. $K(x, t) = \begin{cases} t^2(1-x) & \text{for } t < x \\ x^2(1-t) & \text{for } t > x \end{cases}$
3. $K(x, t) = \begin{cases} \sqrt{t}(1-x) & \text{for } t < x \\ \sqrt{x}(1-t) & \text{for } t > x \end{cases}$



$$4. K(x, t) = \begin{cases} \sqrt{t^3}(1-x) & \text{for } t < x \\ \sqrt{x^3}(1-t) & \text{for } t > x \end{cases}$$

PART - C

38. Let $\lambda_1 < \lambda_2$ be two real characteristic numbers for the following homogeneous integral equation:

$$\varphi(x) = \lambda \int_0^{2\pi} \sin(x+t) \varphi(t) dt;$$

and let $\mu_1 < \mu_2$ be two real characteristic numbers for the following homogeneous integral equation:

$$\psi(x) = \mu \int_0^\pi \cos(x+t) \psi(t) dt.$$

Which of the following statements are true?

1. $\mu_1 < \lambda_1 < \lambda_2 < \mu_2$
2. $\lambda_1 < \mu_1 < \mu_2 < \lambda_2$
3. $|\mu_1 - \lambda_1| = |\mu_2 - \lambda_2|$
4. $|\mu_1 - \lambda_1| = 2|\mu_2 - \lambda_2|$

DECEMBER - 23

PART - B

39. The value of λ for which the integral equation

$$y(x) = \lambda \int_0^1 x^2 e^{x+t} y(t) dt$$

has a non-zero solution, is

- | | |
|-----------------------|-----------------------|
| (1) $\frac{4}{1+e^2}$ | (2) $\frac{2}{1+e^2}$ |
| (3) $\frac{4}{e^2-1}$ | (4) $\frac{2}{e^2-1}$ |

PART - C

40. Consider the following Fredholm integral equation

$$y(x) - 3 \int_0^1 tx y(t) dt = f(x),$$

where $f(x)$ is a continuous function defined on the interval $[0, 1]$. Which of the following choices for $f(x)$ have the property that the above integral equation admits at least one solution?

- | | |
|--------------------------------|--------------------|
| (1) $f(x) = x^2 - \frac{1}{2}$ | (2) $f(x) = e^x$ |
| (3) $f(x) = 2 - 3x$ | (4) $f(x) = x - 1$ |

41. Let y be the solution to the Volterra integral equation

$$y(x) = e^x + \int_0^x \frac{1+x^2}{1+t^2} y(t) dt.$$

Then which of the following statements are true?

- (1) $y(1) = \left(1 + \frac{\pi}{4}\right)e$
- (2) $y(1) = \left(1 + \frac{\pi}{2}\right)e$
- (3) $y(\sqrt{3}) = \left(1 + \frac{3\pi}{4}\right)e^{\sqrt{3}}$
- (4) $y(\sqrt{3}) = \left(1 + \frac{4\pi}{3}\right)e^{\sqrt{3}}$

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PART - B

42. Let u be the solution of the Volterra integral equation

$$\int_0^t \left[\frac{1}{2} + \sin(t-\tau) \right] u(\tau) d\tau = \sin t.$$

Then the value of $u(1)$ is

- | | |
|-------|---------------|
| (1) 0 | (2) 1 |
| (3) 2 | (4) $2e^{-1}$ |

PART - C

43. For $\lambda \in \mathbb{R}$ such that $|\lambda| < \frac{5}{32}$, let $R(x, t, \lambda)$

and u denote the resolvent kernel and the solution, respectively, of the Fredholm integral equation

$$u(x) = x + \frac{\lambda}{2} \int_{-2}^2 (xt + x^2 t^2) u(t) dt.$$

Then which of the following statements are true?

- (1) $R(x, t, \lambda) = \frac{3xt}{3-8\lambda} - \frac{5x^2 t^2}{5-32\lambda}$
- (2) $R(x, t, \lambda) = \frac{3xt}{3-8\lambda} + \frac{5x^2 t^2}{5-32\lambda}$
- (3) $u(1) = -\frac{5}{5-32\lambda}$
- (4) $u(1) = \frac{3}{3-8\lambda}$



44. For $c \in \mathbb{R}$, consider the following Fredholm integral equation

$$y(x) = 1 + x + cx^2 + 2 \int_0^1 (1 - 3xt) y(t) dt.$$

Then the values of c for which the integral equation admits a solution are

- (1) -8 (2) -6
(3) 2 (4) 6

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PART - B

45. Let $\lambda \in \mathbb{R}$, and $K : [0,1] \times [0,1] \rightarrow \mathbb{R}$ be a function such that every solution of the boundary value problem

$$\frac{d^2 u}{dx^2}(x) + \lambda u(x) = 0; \frac{du}{dx}(0) = u(0),$$

$$\frac{du}{dx}(1) = 0$$

satisfies the integral equation

$$u(x) + \lambda \int_0^1 K(x,t) u(t) dt = 0. \text{ Then}$$

$$(1) K(x,t) = \begin{cases} (1+x)(1-t), & 0 \leq x \leq t \leq 1, \\ (1+t)(1-x), & 0 \leq t < x \leq 1 \end{cases}$$

$$(2) K(x,t) = \begin{cases} (-1-x), & 0 \leq x \leq t \leq 1, \\ (-1-t), & 0 \leq t < x \leq 1 \end{cases}$$

$$(3) K(x,t) = \begin{cases} (1-x^2), & 0 \leq x \leq t \leq 1, \\ (1-t^2), & 0 \leq t < x \leq 1 \end{cases}$$

$$(4) K(x,t) = \begin{cases} (1+x)(t-1), & 0 \leq x \leq t \leq 1, \\ (1+t)(x-1), & 0 \leq t < x \leq 1 \end{cases}$$

PART - C

46. The integral equation

$$u(x) = f(x) + \frac{2}{\pi} \int_0^\pi \sin(x-t) u(t) dt \text{ has a}$$

unique solution if

- (1) $f(x) = \cos x$
(2) $f(x) = \cos 5x$
(3) $f(x) = \sin x$
(4) $f(x) = \sin 5x$

47. If u is the solution of the Volterra integral

equation

$$u(x) = 3 + \sin x + \int_0^x \frac{3 + \sin t}{3 + \sin t} u(t) dt, \text{ then}$$

$$(1) u\left(\frac{\pi}{2}\right) = 4e^{\frac{\pi}{2}} \quad (2) u(\pi) = 3e^\pi$$

$$(3) u(-\pi) = 4e^{-\pi} \quad (4) u\left(-\frac{\pi}{2}\right) = 4e^\pi$$

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PART - B

48. If $y(x)$ is the solution of the integral equation

$$y(x) = x^2 + 2 \int_0^1 xty(t) dt,$$

then which of the following statements is true?

$$(1) y(0) + y(1) = \frac{1}{2}$$

$$(2) y(-1) + y(1) = 1$$

$$(3) y'(0) + y'(1) = \frac{3}{2}$$

$$(4) y'(-1) + y'(1) = 3$$

PART - C

49. Let $u(x)$ be the solution to the Volterra integral equation

$$u(x) = x^2 + 4 \int_0^x (t-x)^2 u(t) dt$$

Then which of the following statements are true?

$$1. u(0) = 0$$

$$2. u\left(\frac{2\pi}{\sqrt{3}}\right) = \frac{1}{6} \left(e^{\frac{4\pi}{\sqrt{3}}} - e^{\frac{2\pi}{\sqrt{3}}} \right)$$

$$3. u\left(\frac{\pi}{2\sqrt{3}}\right) = \frac{1}{6} \left(e^{\frac{\pi}{\sqrt{3}}} - \sqrt{3} e^{-\frac{\pi}{2\sqrt{3}}} \right)$$

$$4. u\left(\frac{\pi}{2\sqrt{3}}\right) = \frac{1}{6} \left(e^{\frac{\pi}{\sqrt{3}}} + \sqrt{3} e^{-\frac{\pi}{\sqrt{3}}} \right)$$

50. Let f and K be such that the solution of the initial value problem

$$y'' - 3y' + 2y = 4 \sin(x), \quad y(0) = 1, y'(0) = -2$$

Satisfies the Volterra integral equation



$$y(x) = f(x) + \int_0^x K(x,t)y(t) dt.$$

Then which of the following statements are true?

1. $f'(\pi) = 3$
2. $f(\pi) + f'(\pi) = 4 - \pi$
3. $f(\pi) + f'(\pi) = 2 - \pi$
4. $f(0) + f'(0) = -4$



ANSWERS

- | | | |
|---------------|-------------|-------------|
| 1. (4) | 2. (1,4) | 3. (1,4) |
| 4. (3) | 5. (1,4) | 6. (4) |
| 7. (3) | 8. (2) | 9. (2,4) |
| 10. (1,3) | 11. (4) | 12. (1) |
| 13. (1) | 14. (1,4) | 15. (2,3,4) |
| 16. (2) | 17. (4) | 18. (2) |
| 19. (2) | 20. (1) | 21. (1,4) |
| 22. (2) | 23. (1) | 24. (1,4) |
| 25. (4) | 26. (3,4) | 27. (3) |
| 28. (3) | 29. (1) | 30. (3) |
| 31. (2,4) | 32. (2) | 33. (1,2,3) |
| 34. (3) | 35. | 36. |
| 37. (1) | 38. (1,3) | 39. (3) |
| 40. (1,3) | 41. (2,4) | 42. (1) |
| 43. (2,4) | 44. (1) | 45. (2) |
| 46. (1,2,3,4) | 47. (1,2) | 48. (4) |
| 49. (1,2,3) | 50. (1,2,4) | |