LINEAR ALGEBRA
PREVIOUS YEAR PAPERS

JUNE – 2014

PART - B

1. Let A be a 5×5 matrix with real entries such that the sum of the entries in each row of A is 1. Then the sum of all the entries in A^3 is

1. 3
2. 15
3. 5
4. 125

2. Let J denote a 101 $\times$ 101 matrix with all the entries equal to 1 and let I denote the identity matrix of order 101. Then the determinant of J-I is

1. 101
2. 1
3. 0
4. 100

3. Let $M_{m\times n}(\mathbb{R})$ be the set of all $m\times n$ matrices with real entries. Which of the following statements is correct?

1. There exists $A\in M_{2\times5}(\mathbb{R})$ such that the dimension of the null space of A is 2.
2. There exists $A\in M_{2\times5}(\mathbb{R})$ such that the dimension of the null space of A is 0.
3. There exist $A\in M_{2\times5}(\mathbb{R})$ and $B\in M_{5\times2}(\mathbb{R})$ such that AB is the 2×2 identity matrix.
4. There exists $A\in M_{2\times5}(\mathbb{R})$ whose null space is $\{(x_1, x_2, x_3, x_4, x_5)\in\mathbb{R}^5 : x_1=x_2, x_3=x_4 = x_5\}$

4. For the matrix A as given below, which of them satisfy $A^6=I$?

1. $A=\begin{pmatrix}\cos\frac{\pi}{4} & \sin\frac{\pi}{4} & 0\\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0\\ 0 & 0 & 1\end{pmatrix}$
2. $A=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\frac{\pi}{3} & \sin\frac{\pi}{3}\\ 0 & -\sin\frac{\pi}{3} & \cos\frac{\pi}{3}\end{pmatrix}$
3. $A=\begin{pmatrix}\cos\frac{\pi}{6} & 0 & \sin\frac{\pi}{6}\\ 0 & 1 & 0\\ -\sin\frac{\pi}{6} & 0 & \cos\frac{\pi}{6}\end{pmatrix}$
4. $A=\begin{pmatrix}\cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0\\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0\\ 0 & 0 & 1\end{pmatrix}$

PART – C

5. Let V denote a vector space over a field F and with a basis $B=\{e_1,e_2,\ldots,e_n\}$. Let $x_1, x_2,\ldots, x_n\in F$.
Let
$C=\{x_1e_1, x_1e_1+x_2e_2,\ldots, x_1e_1+x_2e_2+\ldots+x_ne_n\}$.
Then

1. C is linearly independent set implies that $x_i\neq0$ for every i=1,2,…,n.
2. $x_i\neq0$ for every i=1,2,…,n implies that C is linearly independent set.
3. The linear span of C is V implies that $x_i\neq0$ for every i=1,2,…,n.
4. $x_i\neq0$ for every i=1,2,…,n implies that the linear span of C is V.

6. Let V denote the vector space of all polynomials over $\mathbb{R}$ of degree less than or equal to n. Which of the following defines a norm on V?

1. $\|p\|^2=|p(1)|^2+\ldots+|p(n+1)|^2, p\in V$
2. $\|p\|=\sup_{t\in[0,1]}|p(t)|, p\in V$
3. $\|p\|=\int_0^1|p(t)|\,dt, p\in V$
4. $\|p\|=\sup_{t\in[0,1]}|p'(t)|, p\in V$

7. Let u,v,w be vectors in an inner- product space V, satisfying ||u||=||v||=||w||=2 and <u,v>=0, <u,w> = 1, <v,w>= - 1. Then which of the following are true?

1. $\|w+v-u\|=2\sqrt{2}$.

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2. $\left\{\frac{1}{2}u, \frac{1}{2}v\right\}$ forms an orthonormal basis of a two dimensional subspace of V.
3. w and 4u-w are orthogonal to each other.
4. u,v,w are necessarily linearly independent.

8. Let A be a 4×4 matrix over $\mathbb{C}$ such that rank(A)=2 and $A^3= A^2 \neq 0$. Suppose that A is not diagonalizable. Then
1. One of the Jordan blocks of the Jordan canonical form of A is $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.
2. $A^2=A \neq 0$.
3. There exists a vector v such that $Av \neq 0$ but $A^2v = 0$.
4. The characteristic polynomial of A is $x^4 - x^3$.

9. Let $\varphi: \mathbb{R}^2 \to \mathbb{C}$ be the map defined by $\varphi(x, y) = z,$ where z=x+iy. Let f: $\mathbb{C} \to \mathbb{C}$ be the function $f(z)= z^2$ and $F = \varphi^{-1} f\varphi$. Which of the following are correct?
1. The linear transformation $T(x, y) = 2\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ represents the derivative of F at (x,y).
2. The linear transformation $T(x, y) = 2\begin{pmatrix} x & y \\ y & x \end{pmatrix}$ represents the derivative of F at (x,y).
3. The linear transformation T(z)=2z represents the derivative of f at $z \in \mathbb{C}$.
4. The linear transformation T(z)=2z represents the derivative of f only at 0.

10. Let V be the vector space of polynomials over $\mathbb{R}$ of degree less than or equal to n. For $p(x) = a_0 + a_1x + ... + a_nx^n$ in V, define a linear transformation $T: V \to V$ by $(Tp)(x) = a_0 - a_1x + a_2x^2 - \dots + (-1)^n a_n x^n$. Then which of the following are correct?
1. T is one-to-one
2. T is onto
3. T is invertible
4. det T=0

11. Consider a homogeneous system of linear equations Ax=0, where A is an $m\times n$ real matrix and n > m. Then which of the following statements are always true?
1. Ax=0 has a solution.
2. Ax=0 has no non-zero solution.
3. Ax=0 has a non-zero solution.
4. Dimension of the space of all solutions is at least n-m.

DEC – 2014

PART – B

12. Let A,B be $n\times n$ matrices such that $BA+B^2= I - BA^2$, where I is the $n\times n$ identity matrix. Which of the following is always true?
1. A is nonsingular
2. B is nonsingular
3. A+B is nonsingular
4. AB is nonsingular

13. Which of the following matrices has the same row space as the matrix $\begin{pmatrix} 4 & 8 & 4 \\ 3 & 6 & 1 \\ 2 & 4 & 0 \end{pmatrix}$?
1. $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
2. $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
3. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
4. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

14. The determinant of the $n\times n$ permutation matrix $\begin{bmatrix} & & & & & 1 \\ & & & & 1 & \\ & & & \cdot & & \\ & & \cdot & & & \\ & 1 & & & & \\ 1 & & & & & \end{bmatrix}$
1. $(-1)^n$
2. $(-1)^{\left[\frac{n}{2}\right]}$
3. -1
4. 1

15. The determinant $\begin{vmatrix} 1 & 1+x & 1+x+x^2 \\ 1 & 1+y & 1+y+y^2 \\ 1 & 1+z & 1+z+z^2 \end{vmatrix}$ is equal to
1. $(z-y)(z-x)(y-x)$

2. $(x-y)(x-z)(y-z)$
3. $(x-y)^2(y-z)^2(z-x)^2$
4. $(x^2-y^2)(y^2-z^2)(z^2-x^2)$

16. Which of the following matrices is not diagonalizable over $\mathbb{R}$?

1. $\begin{bmatrix}1&1&0\\0&2&0\\0&0&1\end{bmatrix}$ 2. $\begin{bmatrix}1&1&0\\0&2&1\\0&0&3\end{bmatrix}$
3. $\begin{bmatrix}1&1&0\\0&1&0\\0&0&2\end{bmatrix}$ 4. $\begin{bmatrix}1&0&1\\0&2&0\\0&0&3\end{bmatrix}$

17. Let P be a 2×2 complex matrix such that P*P is the identity matrix, where P* is the conjugate transpose of P. Then the eigenvalues of P are
1. real
2. complex conjugates of each other
3. reciprocals of each other
4. of modulus 1

PART – C

18. Let A be a real $n\times n$ orthogonal matrix, that is, $A^tA=AA^t=I_n$, the $n\times n$ identity matrix. Which of the following statements are necessarily true?
1. $\langle Ax, Ay\rangle=\langle x,y\rangle\ \forall x,y\in\mathbb{R}^n$
2. All eigenvalues of A are either +1 or -1.
3. The rows of A form an orthonormal basis of $\mathbb{R}^n$
4. A is diagonalizable over $\mathbb{R}$.

19. Which of the following matrices have Jordan canonical form equal to $\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}$

1. $\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$ 2. $\begin{pmatrix}0&0&1\\0&0&1\\0&0&0\end{pmatrix}$
3. $\begin{pmatrix}0&1&1\\0&0&0\\0&0&0\end{pmatrix}$ 4. $\begin{pmatrix}0&1&1\\0&0&1\\0&0&0\end{pmatrix}$

20. Let A be a 3×4 and b be a 3×1 matrix with integer entries. Suppose that the system Ax=b has a complex solution. Then
1. Ax=b has an integer solution
2. Ax=b has a rational solution
3. The set of real solutions to Ax=0 has a basis consisting of rational solutions.
4. If b≠0, then A has positive rank.

21. Let f be a non-zero symmetric bilinear form on $\mathbb{R}^3$. Suppose that there exists linear transformations $T_i:\mathbb{R}^3\to\mathbb{R}$, i = 1,2 such that for all $\alpha,\beta\in\mathbb{R}^3$, $f(\alpha,\beta)=T_1(\alpha)T_2(\beta)$. Then
1. rank f=1
2. dim $\{\beta\in\mathbb{R}^3 : f(\alpha,\beta)=0 \text{ for all } \alpha\in\mathbb{R}^3\}=2$
3. f is positive semi-definite or negative semi- definite.
4. $\{\alpha: f(\alpha,\alpha)=0\}$ is a linear subspace of dimension 2

22. The matrix $A=\begin{pmatrix}5&9&8\\1&8&2\\9&1&0\end{pmatrix}$ satisfies
1. A is invertible and the inverse has all integer entries.
2. det(A) is odd.
3. det(A) is divisible by 13.
4. det(A) has at least two prime divisors.

23. Let A be 5×5 matrix and let B be obtained by changing one element of A. Let r and s be the ranks of A and B respectively. Which of the following statements is/are correct?
1. $s\le r+1$ 2. $r-1\le s$
3. $s=r-1$ 4. $s\ne r$

24. Let $M_n(K)$ denote the space of all $n\times n$ matrices with entries in a field K. Fix a non-singular matrix $A=(A_{ij})\in M_n(K)$, and consider the linear map $T:M_n(K)\to M_n(K)$ given by T(X)=AX. Then
1. trace (T)=n$\sum_{i=1}^n A_{ii}$
2. trace(T) $=\sum_{i=1}^n\sum_{j=1}^n A_{ij}$
3. rank of T is n^2
4. T is non – singular

25. For arbitrary subspaces U,V and W of a finite dimensional vector space, which of the following hold
1. $U\cap(V+W)\subset U\cap V+U\cap W$
2. $U\cap(V+W)\supset U\cap V+U\cap W$

3. $(U \cap V) + W \subset (U+W) \cap (V+W)$
4. $(U \cap V) + W \supset (U+W) \cap (V+W)$

26. Let A be 4×7 real matrix and B be a 7×4 real matrix such that $AB = I_4$, where I_4 is the 4×4 identity matrix. Which of the following is/are always true?
1. rank (A)=4
2. rank (B)=7
3. nullity (B)=0
4. BA=I_7, where I_7 is the 7×7 identity matrix

27. Let ℝ[x] denote the vector space of all real polynomials. Let D : ℝ[x] → ℝ [x] denote the map $Df = \frac{df}{dx}, \forall f$. Then,
1. D is one-one
2. D is onto
3. There exists E : ℝ[x] → ℝ[x] so that $D(E(f)) = f, \forall f$.
4. There exists E: ℝ[x] → ℝ[x] so that $E(D(f)) = f, \forall f$.

28. Which of the following are eigenvalues of the matrix $\begin{pmatrix} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \end{pmatrix}$?

1. +1   2. -1
3. +i   4. –i

29. Let $A = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$, where $x, y \in \mathbb{R}$ such that $x^2 + y^2 = 1$. Then we must have
1. For any $n \geq 1$, $A^n = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ where $x = \cos(\theta/n), y = \sin(\theta/n)$
2. $tr(A) \neq 0$
3. $A^t = A^{-1}$
4. A is similar to a diagonal matrix over ℂ

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PART – B

30. Let V be the space of twice differentiable functions on ℝ satisfying $f'' - 2f' + f = 0$. Define T: V → $\mathbb{R}^2$ by $T(f) = (f'(0), f(0))$. Then T is
1. one –to-one and onto
2. one-to-one but not onto
3. onto but not one-to-one
4. neither one-to-one nor onto

31. The row space of a 20×50 matrix A has dimension 13. What is the dimension of the space of solutions of Ax = 0?
1. 7   2. 13
3. 33   4. 37

32. Let A,B be $n \times n$ matrices. Which of the following equals trace(A^2B^2)?
1. $(\text{trace}(AB))^2$ 2. trace(AB^2A)
3. trace($(AB)^2$)   4. trace(BABA)

33. Given a 4×4 real matrix A, let T:$\mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation defined by Tv = Av, where we think of $\mathbb{R}^4$ as the set of real 4×1 matrices. For which choices of A given below, do Image(T) and Image(T^2) have respective dimensions 2 and 1?
($*$ denotes a non zero entry)

1. $A = \begin{bmatrix} 0&0&*&* \\ 0&0&*&* \\ 0&0&0&* \\ 0&0&0&0 \end{bmatrix}$

2. $A = \begin{bmatrix} 0&0&*&0 \\ 0&0&*&0 \\ 0&0&0&* \\ 0&0&0&* \end{bmatrix}$

3. $A = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&* \\ 0&0&*&0 \end{bmatrix}$

4. $A = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&*&* \\ 0&0&*&* \end{bmatrix}$

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34. Let T be a 4×4 real matrix such that $T^4 = 0$. Let $k_i = \dim \operatorname{Ker} T^i$ for $1 \le i \le 4$. Which of the following is NOT a possibility for the sequence $k_1 \le k_2 \le k_3 \le k_4$?
1. $3 \le 4 \le 4 \le 4$.
2. $1 \le 3 \le 4 \le 4$.
3. $2 \le 4 \le 4 \le 4$.
4. $2 \le 3 \le 4 \le 4$.

35. Which of the following is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$?

(a) $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x+y \end{pmatrix}$ (b) $g\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}$

(c) $h\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z-x \\ x+y \end{pmatrix}$

1. Only f.
2. Only g.
3. Only h.
4. all the transformations f,g and h.

PART – C

36. Let A be an m×n matrix of rank n with real entries. Choose the correct statement.
1. Ax = b has a solution for any b.
2. Ax = 0 does not have a solution.
3. If Ax = b has a solution, then it is unique.
4. y'A = 0 for some nonzero y, where y' denotes the transpose of the vector y.

37. Let F: $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be the function F(x,y)=<Ax,y> where $\langle , \rangle$ is the standard inner product of $\mathbb{R}^n$ and A is a n×n real matrix. Here D denotes the total derivative. Which of the following statements are correct?
1. $(DF(x, y))(u, v) = \langle Au, y\rangle + \langle Ax, v\rangle$
2. (DF(x,y))(0,0) = 0.
3. DF(x,y) may not exist for some $(x,y) \in \mathbb{R}^n \times \mathbb{R}^n$
4. DF(x,y) does not exist at (x,y) = (0,0).

38. Let $f:\mathbb{R}^n \to \mathbb{R}^n$ be a continous function such that $\int_{R^n} |f(x)|\, dx < \infty$. Let A be a real n×n invertible matrix and for x,y $\in \mathbb{R}^n$, let $\langle x, y\rangle$ denote the standard inner product in R^n. Then $\int_{R^n} f(Ax) e^{i\langle y,x\rangle} dx =$

1. $\int_{R^n} f(x) e^{i\langle (A^{-1})^T y, x\rangle} \frac{dx}{|\det A|}$
2. $\int_{R^n} f(x) e^{i\langle A^T y, x\rangle} \frac{dx}{|\det A|}$
3. $\int_{R^n} f(x) e^{i\langle (A^T)^{-1} y, x\rangle} dx$
4. $\int_{R^n} f(x) e^{i\langle A^{-1} y, x\rangle} \frac{dx}{|\det A|}$

39. Let S be the set of 3×3 real matrices A with $A^T A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Then the set S contains
1. a nilpotent matrix.
2. a matrix of rank one.
3. a matrix of rank two.
4. a non-zero skew-symmetric matrix.

40. An n×n complex matrix A satisfies $A^k = I_n$, the n×n identity matrix, where k is a positive integer > 1. Suppose 1 is not an eigenvalue of A. Then which of the following statements are necessarily true?
1. A is diagonalizable.
2. $A+A^2+\ldots+A^{k-1} = O$, the n×n zero matrix
3. $\operatorname{tr}(A) + \operatorname{tr}(A^2) + \ldots + \operatorname{tr}(A^{k-1}) = -n$
4. $A^{-1} + A^{-2} + \ldots + A^{-(k-1)} = -I_n$

41. Let S: $\mathbb{R}^n \to \mathbb{R}^n$ be given by S(v) = αv for a fixed $\alpha \in \mathbb{R}$, α ≠ 0.

Let T:$\mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation such that B = $\{v_1, \ldots, v_n\}$ is a set of linearly independent eigen vectors of T. Then
1. The matrix of T with respect to B is diagonal.
2. The matrix of (T - S) with respect to B is diagonal.
3. The matrix of T with respect to B is not necessarily diagonal, but is upper triangular.
4. The matrix of T with respect to B is diagonal but the matrix of (T-S) with respect to B is not diagonal.

42. Let $p_n(x) = x^n$ for $x \in \mathbb{R}$ and let $\wp$ = span $\{p_0, p_1, p_2, \ldots\}$. Then
1. $\wp$ is the vector space of all real valued continuous functions on $\mathbb{R}$.
2. $\wp$ is a subspace of all real valued continuous functions on $\mathbb{R}$.

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3. $\{p_0, p_1, p_2, \ldots\}$ is a linearly independent set in vector space of all continuous functions on $\mathbb{R}$.
4. Trigonometric functions belong to $\wp$.

43. Let $A = \begin{bmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{bmatrix}$ be a 3×3 matrix where a,b,c,d are integers. Then, we must have:

1. If $a \neq 0$, there is a polynomial $p \in \mathbb{Q}[x]$ such that p(A) is the inverse of A.
2. For each polynomial $q \in \mathbb{Z}[x]$, the matrix $q(A) = \begin{bmatrix} q(a) & q(b) & q(c) \\ 0 & q(a) & q(d) \\ 0 & 0 & q(a) \end{bmatrix}$
3. If $A^n = O$ for some positive integer n, then $A^3 = O$.
4. A commutes with every matrix of the form $\begin{bmatrix} a' & 0 & c' \\ 0 & a' & 0 \\ 0 & 0 & a' \end{bmatrix}$.

44. Which of the following are subspaces of vector space $\mathbb{R}^3$?
1. $\{(x,y,z) : x + y = 0\}$
2. $\{(x,y,z) : x - y = 0\}$
3. $\{(x,y,z) : x + y = 1\}$
4. $\{(x,y,z) : x - y = 1\}$

45. Consider non-zero vector spaces V_1, V_2, V_3, V_4 and linear transformations $T_1: V_1 \to V_2$, $T_2: V_2 \to V_3$, $T_3: V_3 \to V_4$ such that $\text{Ker}(T_1) = \{0\}$, $\text{Range}(T_1) = \text{Ker}(T_2)$, $\text{Range}(T_2) = \text{Ker}(T_3)$, $\text{Range}(T_3) = V_4$. Then

1. $\sum_{i=1}^{4} (-1)^i \dim V_i = 0$
2. $\sum_{i=2}^{4} (-1)^i \dim V_i > 0$
3. $\sum_{i=1}^{4} (-1)^i \dim V_i < 0$
4. $\sum_{i=1}^{4} (-1)^i \dim V_i \neq 0$

46. Let A be an invertible 4×4 real matrix. Which of the following are NOT true?
1. Rank A = 4.
2. For every vector $b \in \mathbb{R}^4$, $Ax = b$ has exactly one solution.
3. dim (nullspace A) ≥ 1.
4. 0 is an eigenvalue of A.

47. Let $\underline{u}$ be a real n×1 vector satisfying $\underline{u}'\underline{u}=1$, where $\underline{u}'$ is the transpose of $\underline{u}$. Define $A = I - 2\underline{u}\underline{u}'$ where I is the n^{th} order identity matrix. Which of the following statements are true?

1. A is singular
2. $A^2 = A$
3. Trace(A) = n-2
4. $A^2 = I$

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<u>PART – B</u>

48. Let S denote the set of all the prime numbers p with the property that the matrix $\begin{bmatrix} 91 & 31 & 0 \\ 29 & 31 & 0 \\ 79 & 23 & 59 \end{bmatrix}$ has an inverse in the field $\mathbb{Z}/p\mathbb{Z}$. Then

1. S = {31}
2. S = {31, 59}
3. S = {7, 13, 59}
4. S is infinite

49. For a positive integer n, let P_n denote the vector space of polynomials in one variable x with real coefficients and with degree ≤ n. Consider the map $T: P_2 \to P_4$ defined by $T(p(x)) = p(x^2)$. Then
1. T is a linear transformation and dim range (T) = 5.
2. T is a linear transformation and dim range (T) = 3.
3. T is a linear transformation and dim range (T) = 2.
4. T is not a linear transformation.

50. Let A be a real 3 x 4 matrix of rank 2. Then the rank of A^tA, where A^t denotes the transpose of A, is:
1. exactly 2
2. exactly 3
3. exactly 4
4. at most 2 but not necessarily 2

51. Consider the quadratic form $Q(v) = v^tAv$, where $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, v = (x, y, z, w)$

Then
1. Q has rank 3.
2. $xy + z^2 = Q(Pv)$ for some invertible 4 x 4 real matrix P

3. $xy + y^2 + z^2 = Q(Pv)$ for some invertible 4 x 4 real matrix P
4. $x^2 + y^2 - zw = Q(Pv)$ for some invertible 4 x 4 real matrix P.

52. If A is a 5 x 5 real matrix with trace 15 and if 2 and 3 are eigenvalues of A, each with algebraic multiplicity 2, then the determinant of A is equal to

1. 0 2. 24
3. 120 4. 180

53. Let $A \neq I_n$ be an n x n matrix such that $A^2 = A$, where I_n is the identity matrix of order n. Which of the following statements is false?

1. $(I_n - A)^2 = I_n - A$.
2. Trace (A) = Rank (A).
3. Rank (A) + Rank $(I_n - A)$ = n.
4. The eigenvalues of A are each equal to 1.

PART – C

54. Let A and B be n x n matrices over $\mathbb{C}$. Then,

1. AB and BA always have the same set of eigenvalues.
2. If AB and BA have the same set of eigen-values then AB = BA.
3. If A^{-1} exists then AB and BA are similar.
4. The rank of AB is always the same as the rank of BA.

55. Let A be an m x n real matrix and $b \in \mathbb{R}^m$ with $b \neq 0$.

1. The set of all real solutions of Ax = b is a vector space.
2. If u and v are two solutions of Ax = b, then $\lambda u + (1 - \lambda)v$ is also a solution of Ax = b, for any $\lambda \in \mathbb{R}$.
3. For any two solutions u and v of Ax = b, the linear combination $\lambda u + (1 - \lambda)$ v is also a solution of Ax = b only when $0 \leq \lambda \leq 1$.
4. If rank of A is n, then Ax = b has at most one solution.

56. Let A be an n x n matrix over $\mathbb{C}$ such that every nonzero vector of $\mathbb{C}^n$ is an eigenvector of A. Then.

1. All eigenvalues of A are equal.
2. All eigenvalues of A are distinct.
3. $A = \lambda$ I for some $\lambda \in \mathbb{C}$, where I is the n x n identity matrix.
4. If χ_A and m_A denote the characteristic polynomial and the minimal polynomial respectively, then $\chi_A = m_A$.

57. Consider the matrices $A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Then

1. A and B are similar over the field of rational numbers $\mathbb{Q}$.
2. A is diagonalizable over the field of rational numbers $\mathbb{Q}$.
3. B is the Jordan canonical form of A.
4. The minimal polynomial and the characteristic polynomial of A are the same

58. Let V be a finite dimensional vector space over $\mathbb{R}$. Let $T : V \to V$ be a linear transformation such that rank (T^2) = rank (T). Then,

1. Kernel (T^2) = Kernel (T).
2. Range (T^2) = Range (T).
3. Kernel (T) $\cap$ Range (T) = {0}.
4. Kernel $(T^2) \cap$ Range (T^2) = {0}.

59. Let V be the vector space of polynomials over $\mathbb{R}$ of degree less than or equal to n. For $p(x) = a_0 + a_1x + \ldots + a_nx^n$ in V, define a linear transformation $T:V\to V$ by $(Tp)(x) = a_n + a_{n-1}x + \ldots + a_0x^n$. Then

1. T is one to one. 2. T is onto.
3. T is invertible. 4. det T = $\pm$ 1.

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PART – B

60. Given a $n \times n$ matrix B define e^B by $e^B = \sum_{j=0}^{\infty} \frac{B^j}{j!}$

Let p be the characteristic polynomial of B. Then the matrix $e^{p(B)}$ is

1. $I_{n\times n}$ 2. $0_{n\times n}$
3. $eI_{n\times n}$ 4. $\pi I_{n\times n}$

61. Let A be a $n \times m$ matrix and b be a $n \times 1$ vector (with real entries). Suppose the equation Ax=b, $x \in R^m$ admits a unique solution. Then we can conclude that

1. $m \geq n$ 2. $n \geq m$
3. $n = m$ 4. $n > m$

62. Let V be the vector space of all real polynomials of degree $\leq$ 10. Let $Tp(x) = p'(x)$ for $p \in V$ be a linear transformation from V to V. Consider the basis $\{1, x, x^2, \ldots, x^{10}\}$ of V. Let A be the matrix of T with respect to this basis. Then
1. Trace A=1
2. det A=0
3. there is no $m \in \mathbb{N}$ such that $A^m=0$
4. A has a non zero eigenvalue

63. Let $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in \mathbb{R}^3$ be linearly independent. Let $\delta_1 = x_2y_3 - y_2x_3$, $\delta_2 = x_1y_3 - y_1x_3$, $\delta_3 = x_1y_2 - y_1x_2$. If V is the span of x,y then
1. $V = \{(u,v,w) : \delta_1 u - \delta_2 v + \delta_3 w = 0\}$
2. $V = \{(u,v,w) : -\delta_1 u + \delta_2 v + \delta_3 w = 0\}$
3. $V = \{(u,v,w) : \delta_1 u + \delta_2 v - \delta_3 w = 0\}$
4. $V = \{(u,v,w) : \delta_1 u + \delta_2 v + \delta_3 w = 0\}$

64. Let A be a $n \times n$ real symmetric non-singular matrix. Suppose there exists $x \in \mathbb{R}^n$ such that $x'Ax < 0$. Then we can conclude that
1. det (A)<0
2. B = -A is positive definite
3. $\exists y \in \mathbb{R}^n; y'A^{-1}y < 0$
4. $\forall y \in \mathbb{R}^n;\ y'A^{-1}y < 0$

65. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Let $f : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(v,w) = w^T Av$.
Pick the correct statement from below:
1. There exists an eigenvector v of A such that Av is perpendicular to v
2. The set $\{v \in \mathbb{R}^2 | f(v,v) = 0\}$ is a nonzero subspace of $\mathbb{R}^2$
3. If $v, w \in \mathbb{R}^2$ are non zero vectors such that $f(v,v) = 0 = f(w,w)$, then v is a scalar multiple of w.
4. For every $v \in \mathbb{R}^2$, there exists a non zero $w \in \mathbb{R}^2$ such that $f(v,w) = 0$.

PART – C

66. Let V be the vector space of all complex polynomials p with deg p ≤ n. Let T :V → V be the map (Tp) (x) = $p'(1)$, x∈ℂ. Which of the following are correct?

1. dim Ker T = n. 2.dim range T = 1.
3. dim Ker T = 1. 4. dim range T=n+1.

67. Let A be an n ×n real matrix. Pick the correct answer(s) from the following
1. A has at least one real eigenvalue.
2. For all nonzero vectors v,w∈$\mathbb{R}^n$, $(Aw)^T(Av) > 0$.
3. Every eigenvalue of A^TA is a non negative real number.
4. I + A^TA is invertible.

68. Let T be a n × n matrix with the property $T^n=0$. Which of the following is/are true?
1. T has n distinct eigenvalues
2. T has one eigenvalue of multiplicity n
3. 0 is an eigenvalue of T.
4. T is similar to a diagonal matrix.

69. Let V = {f: [0,1] → ℝ|f is a polynomial of degree less than or equal to n}. Let $f_j(x) = x^j$ for 0 ≤ j ≤ n and let A be the (n+1) × (n + 1) matrix given by $a_{ij} = \int_0^1 f_i(x) f_j(x) dx$.
Then which of the following is/are true?
1. dim V = n.
2. dim V > n.
3. A is nonnegative definite, i.e., for all v∈ $\mathbb{R}^n$, $\langle Av, v \rangle \geq 0$.
4. det A > 0.

70. Consider the real vector space V of polynomials of degree less than or equal to d. For p∈V define $||p||_k$ = max {|p(0)|, |p′(0)|,..., $|p^{(k)}(0)|$}, where $p^{(t)}(0)$ is the i^{th} derivative of p evaluated at 0, Then $||p||_k$ defines a norm on V if and only if
1. k ≥ d – 1 2. k < d
3. k ≥ d 4. k < d – 1

71. Let A, B be n×n real matrices such that det A > 0 and det B < 0. For 0 ≤ t ≤ 1. Consider C(t) = t A + (1- t)B. Then
1. C(t) is invertible for each t∈[0,1].
2. There is a t_0 ∈ (0,1) such that C(t_0) is not invertible.
3. C(t) is not invertible for each t∈[0,1].
4. C(t) is invertible for only finitely many t∈[0,1].

72. Let $\{a_1, \ldots, a_n\}$ and $\{b_1, \ldots, b_n\}$ be two bases of $\mathbb{R}^n$. Let P be n×n matrix with real entries such that $Pa_i=b_i$ i=1,2,…,n. Suppose that every eigenvalue of P is either –1 or 1. Let Q = I + 2P.

Then which of the following statements are true?
1. $\{a_i + 2b_i \mid i=1,2,\ldots,n\}$ is also a basis of V.
2. Q is invertible.
3. Every eigenvalue of Q is either 3 or -1.
4. det Q > 0 if det P > 0.

73. Let A be an n × n matrix with real entries. Define $\langle x, y\rangle_A = \langle Ax, Ay\rangle, x, y \in \mathbb{R}^n$.
Then $\langle x, y\rangle_A$ defines an inner-product if and only if
1. ker A= {0}.
2. rank A = n.
3. All eigenvalues of A are positive.
4. All eigenvalues of A are non-negative.

74. Suppose $\{v_1, \ldots, v_n\}$ are unit vectors in $\mathbb{R}^n$ such that $\|v\|^2 = \sum_{i=1}^{n} \left|\langle v_i, v\rangle\right|^2 \; \forall v \in \mathbb{R}^n$
Then decide the correct statements in the following
1. $v_1,\ldots,v_n$ are mutually orthogonal
2. $\{v_1,\ldots, v_n\}$ is a basis for $\mathbb{R}^n$
3. $v_1,\ldots,v_n$ are not mutually orthogonal
4. Atmost n – 1 of the elements in the set $\{v_1,\ldots, v_n\}$can be orthogonal.

DEC – 2016

PART – B

75. The matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ is
1. positive definite.
2. non-negative definite but not positive definite.
3. negative definite
4. neither negative definite nor positive definite

76. Which of the following subsets of $\mathbb{R}^4$ is a basis of $\mathbb{R}^4$?
B_1 = {(1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1)}
B_2 = {(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)}
B_3 = {(1,2,0,0), (0,0,1,1), (2,1,0,0), (-5,5,0,0)}
1. B_1 and B_2 but not B_3
2. B_1, B_2 and B_3
3. B_1 and B_3 but not B_2
4. Only B_1

77. Let $D_1 = \det\begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix}$ and
$D_2 = \det\begin{pmatrix} -x & a & -p \\ y & -b & q \\ z & -c & r \end{pmatrix}$. Then
1. $D_1 = D_2$
2. $D_1 = 2D_2$
3. $D_1 = -D_2$
4. $2D_1 = D_2$

78. Consider the matrix $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$,
where $\theta = \frac{2\pi}{31}$ Then A^{2015} equals
1. A
2. I
3. $\begin{pmatrix} \cos 13\theta & \sin 13\theta \\ -\sin 13\theta & \cos 13\theta \end{pmatrix}$
4. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

79. Let J denote the matrix of order n × n with all entries 1 and let B be a (3n) × (3n) matrix given by $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$
Then the rank of B is
1. 2n
2. 3n – 1
3. 2
4. 3

80. Which of the following sets of functions from $\mathbb{R}$ to $\mathbb{R}$ is a vector space over $\mathbb{R}$?
$S_1 = \{f \mid \lim_{x\to 3} f(x) = 0\}$
$S_2 = \left\{g \,\middle|\, \lim_{x\to 3} g(x) = 1\right\}$
$S_3 = \left\{h \,\middle|\, \lim_{x\to 3} h(x)\, exists\right\}$
1. Only S_1
2. Only S_2
3. S_1 and S_3 but not S_2
4. All the three are vector spaces

81. Let A be an n × m matrix with each entry equal to +1, -1 or 0 such that every column has exactly one +1 and exactly one -1. We can conclude that

1. Rank A $\le$ n – 1 2. Rank A = m
3. n $\le$ m 4. n – 1 $\le$ m

82. What is the number of non-singular 3 $\times$ 3 matrices over F_2, the finite field with two elements?
1. 168 2. 384
3. 2^3 4. 3^2

PART – C

83. Let A=[a_{ij}] be an $n\times n$ matrix such that a_{ij} is an integer for all i.j. Let AB = I with B= [b_{ij}] (where I is the identity matrix). For a square matrix C, det C denotes its determinant. Which of the following statements is true?
1. If det A= 1 then det B=1.
2. A sufficient condition for each b_{ij} to be an integer is that det A is an integer.
3. B is always an integer matrix.
4. A necessary condition for each b_{ij} to be an integer is det A $\in$ {-1, +1}.

84. Let $A=\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and let α_n and β_n denote the two eigenvalues of A^n such that $|\alpha_n| \ge |\beta_n|$. Then
1. $\alpha_n \to\infty$ as $n\to\infty$
2. $\beta_n \to 0$ as $n\to\infty$
3. β_n is positive if n is even.
4. β_n is negative if n is odd.

85. Let M_n denote the vector space of all $n\times n$ real matrices. Among the following subsets of M_n, decide which are linear subspaces.
1. $V_1 = \{A\in M_n : A \text{ is nonsingular}\}$
2. $V_2 = \{A\in M_n : \det(A) =0\}$
3. $V_3 = \{A\in M_n : \text{trace}(A) = 0\}$
4. $V_4 = \{BA: A\in M_n \}$, where B is some fixed matrix in M_n.

86. If P and Q are invertible matrices such that PQ = - QP, then we can conclude that
1. Tr(P) = Tr(Q)=0 2. Tr(P) = Tr(Q)=1
3. Tr(P) = - Tr(Q) 4. Tr(P) $\ne$ Tr(Q)

87. Let n be an odd number ≥ 7. Let A=[a_{ij}] be an $n\times n$ matrix with $a_{i,i+1}$=1 for all i=1,2,..., n-1 and $a_{n,1}$=1. Let a_{ij}=0 for all the other pairs (i,j). Then we can conclude that
1. A has 1 as an eigenvalue.
2. A has -1 as an eigenvalue.
3. A has at least one eigenvalue with multiplicity ≥ 2.
4. A has no real eigenvalues.

88. Let W_1, W_2, W_3 be three distinct subspaces of $\mathbb{R}^{10}$ such that each W_i has dimension 9. Let $W=W_1\cap W_2\cap W_3$. Then we can conclude that
1. W may not be a subspace of $\mathbb{R}^{10}$
2. dim W ≤ 8
3. dim W ≥ 7
4. dim W ≤ 3

89. Let A be a real symmetric matrix. Then we can conclude that
1. A does not have 0 as an eigenvalue
2. All eigenvalues of A are real
3. If A^{-1} exists, then A^{-1} is real and symmetric
4. A has at least one positive eigenvalue

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PART – B

90. Let A be a 4 $\times$ 4 matrix. Suppose that the null space N(A) of A is
$\{(x, y, z, w) \in \mathbb{R}^4 : x+y+z = 0, x+y+w = 0\}$. Then
1. dim (column space (A)) = 1
2. dim (column space (A)) = 2
3. rank (A) = 1
4. S = {(1,1,1,0), (1,1,0,1)} is a basis of N(A)

91. Let A and B be real invertible matrices such that AB = -BA. Then
1. Trace (A) = Trace (B) = 0
2. Trace (A) = Trace (B) = 1
3. Trace (A) = 0, Trace (B) = 1
4. Trace (A) = 1, Trace (B) = 0

92. Let A be an n $\times$ n self-adjoint matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$.
Let $\|X\|_2=\sqrt{|x_1|^2+\ldots+|x_n|^2}$ for $X=(x_1,\ldots,x_n)\in\mathbb{C}^n$.
If $p(A)=a_0I+a_1A+\ldots+a_nA^n$ then $\sup_{\|X\|_2=1}\|p(A)X\|_2$ is equal to
1. $\max\{a_0+a_1\lambda_j+\ldots+a_n\lambda_j^n : 1\le j\le n\}$
2. $\max\{|a_0+a_1\lambda_j+\ldots+a_n\lambda_j^n| : 1\le j\le n\}$

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3. $\min\{ a_0 + a_1\lambda_j + ... + a_n\lambda_j^n : 1 \le j \le n\}$
4. $\min\{| a_0 + a_1\lambda_j + ... + a_n\lambda_j^n | : 1 \le j \le n\}$

93. Let $p(x) = \alpha x^2 + \beta x + \gamma$ be a polynomial, where $\alpha, \beta, \gamma \in \mathbb{R}$. Fix $x_0 \in \mathbb{R}$. Let $S = \{(a,b,c) \in \mathbb{R}^3 : p(x) = a(x - x_0)^2 + b(x - x_0) + c$ for all $x \in \mathbb{R}\}$.
Then the number of elements in S is
1. 0
2. 1
3. strictly greater than 1 but finite
4. infinite

94. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ and I be the 3×3 identity matrix. If $6A^{-1} = aA^2 + bA + cI$ for $a, b, c \in \mathbb{R}$ then (a,b,c) equals
1. (1, 2, 1)
2. (1, -1, 2)
3. (4, 1, 1)
4. (1, 4, 1)

95. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 5 \\ 2 & 5 & -3 \end{bmatrix}$.
Then the eigenvalues of A are
1. -4, 3, -3
2. 4, 3, 1
3. $4, -4 \pm \sqrt{13}$
4. $4, -2 \pm 2\sqrt{7}$

PART – C

96. Consider the vector space V of real polynomials of degree less than or equal to n. Fix distinct real numbers $a_0, a_1, ..., a_k$. For $p \in V$, $\max\{|p(a_j)| : 0 \le j \le k\}$ defines a norm on V
1. only if k < n
2. only if k ≥ n
3. if k+1≤n
4. if k ≥ n+1

97. Let V be the vector space of polynomials of degree at most 3 in a variable x with coefficient in $\mathbb{R}$. Let T = d/dx be the linear transformation of V to itself given by differentiation. Which of the following are correct?
1. T is invertible
2. 0 is an eigenvalue of T
3. There is a basis with respect to which the matrix of T is nilpotent.
4. The matrix of T with respect to the basis $\{1, 1+x, 1+x+x^2, 1+x+x^2+x^3\}$ is diagonal

98. Let m,n,r be natural numbers. Let A be $m\times n$ matrix with real entries such that $(AA^t)^r = I$, where I is the $m\times m$ identity matrix and A^t is the transpose of the matrix A. We can conclude that
1. m=n
2. AA^t is invertible
3. $A^t A$ is invertible
4. if m=n, then A is invertible

99. Let A be an $n\times n$ real matrix with $A^2 = A$. Then
1. the eigenvalues of A are either 0 or 1
2. A is a diagonal matrix with diagonal entries 0 or 1
3. rank(A) = trace (A)
4. rank $(I - A)$ = trace $(I - A)$

100. For any $n\times n$ matrix B, let N(B) = $\{X \in \mathbb{R}^n : BX = 0\}$ be the null space of B. Let A be a 4×4 matrix with dim(N($A - 2I$))=2, dim (N($A - 4I$))=1 and rank (A) = 3. Then
1. 0,2 and 4 are eigenvalues of A
2. determinant (A) = 0
3. A is not diagonalizable
4. trace (A) = 8

101. Which of the following 3×3 matrices are diagonalizable over $\mathbb{R}$?
1. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$
2. $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$
4. $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

102. Let H be a real Hilbert space and $M \subseteq H$ be a closed linear subspace. Let $x_0 \in H\backslash M$. Let $y_0 \in M$ be such that $||x_0 - y_0|| = \inf\{||x_0 - y|| : y \in M\}$. Then
1. such a y_0 is unique
2. $x_0 \perp M$
3. $y_0 \perp M$
4. $x_0 - y_0 \perp M$

103. Let $A=\begin{bmatrix}3&1&2\\1&2&3\\2&3&1\end{bmatrix}$ and $Q(X)=X^tAX$ for $X\in\mathbb{R}^3$. Then
1. A has exactly two positive eigenvalues
2. all the eigenvalues of A are positive
3. $Q(X)\geq 0$ for all $X\in\mathbb{R}^3$
4. $Q(X)<0$ for some $X\in\mathbb{R}^3$

104. Consider the matrix
$$A(x)=\begin{pmatrix}1+x^2&7&11\\3x&2x&4\\8x&17&13\end{pmatrix};x\in\mathbb{R}.$$ Then
1. A(x) has eigenvalue 0 for some $x\in\mathbb{R}$
2. 0 is not an eigenvalue of A(x) for any $x\in\mathbb{R}$
3. A(x) has eigenvalue 0 for all $x\in\mathbb{R}$
4. A(x) is invertible for every $x\in\mathbb{R}$

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PART – B

105. Let A be a real symmetric matrix and B = I + iA, where i^2 = -1. Then
1. B is invertible if and only if A is invertible
2. all eigenvalues of B are necessarily real
3. B – I is necessarily invertible
4. B is necessarily invertible

106. Let $A=\begin{bmatrix}0&1\\-1&1\end{bmatrix}$. Then the smallest positive integer n such that A^n = I is
1. 1 2. 2 3. 4 4. 6

107. Let $A=\begin{bmatrix}1&-1&1\\1&1&1\\2&3&\alpha\end{bmatrix}$ and $b=\begin{bmatrix}1\\3\\\beta\end{bmatrix}$. Then the system AX = b over the real numbers has
1. no solution whenever $\beta\neq 7$.
2. an infinite number of solutions whenever $\alpha\neq 2$.
3. an infinite number of solutions if $\alpha = 2$ and $\beta\neq 7$
4. a unique solution if $\alpha\neq 2$

108. Let $A=\begin{bmatrix}1&2\\4&3\end{bmatrix}\in M_2(\mathbb{R})$ and $\phi:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ be the bilinear map defined by ϕ (v, w) = v^TAw. Choose the correct statement from below:
1. $\phi(v, w) = \phi(w, v)$ for all v, w $\in\mathbb{R}^2$
2. there exists nonzero v $\in\mathbb{R}^2$ such that ϕ (v, w) = 0 for all w $\in\mathbb{R}^2$
3. there exists a 2 × 2 symmetric matrix B such that ϕ (v, v) = v^TBv for all v $\in\mathbb{R}^2$
4. the map $\psi:\mathbb{R}^4\to\mathbb{R}$ defined by $\psi\left(\begin{bmatrix}v_1\\v_2\\w_1\\w_2\end{bmatrix}\right)=\phi\left(\begin{bmatrix}v_1\\v_2\end{bmatrix},\begin{bmatrix}w_1\\w_2\end{bmatrix}\right)$ is linear

PART – C

109. Let A be an m×n matrix with rank r. If the linear system AX=b has a solution for each $b\in\mathbb{R}^m$, then
1. m=r
2. the column space of A is a proper subspace of $\mathbb{R}^m$
3. the null space of A is a non-trivial subspace of $\mathbb{R}^n$ whenever m=n
4. m≥n implies m=n

110. Let $M=\{A=\begin{pmatrix}a&b\\c&d\end{pmatrix}\mid a,b,c,d\in\mathbb{Z}$ and the eigenvalues of A are in $\mathbb{Q}\}$. Then
1. M is empty
2. $M=\{\begin{pmatrix}a&b\\c&d\end{pmatrix}\mid a,b,c,d\in\mathbb{Z}\}$
3. If A∈M then the eigenvalues of A are in $\mathbb{Z}$
4. If A,B∈M are such that AB=1 then det A∈(+1,-1)

111. Let A be a 3×3 matrix with real entries. Identify the correct statements.
1. A is necessarily diagonalizable over $\mathbb{R}$
2. If A has distinct real eigenvalues then it is diagonalizable over $\mathbb{R}$
3. If A has distinct eigenvalues then it is diagonalizable over $\mathbb{C}$

4. If all eigenvalues of A are non-zero then it is diagonalizable over $\mathbb{C}$

112. Let V be the vector space over $\mathbb{C}$ of all polynomials in a variable X of degree at most 3. Let $D:V\to V$ be the linear operator given by differentiation with respect to X. Let A be the matrix of D with respect to some basis for V. Which of the following are true?
1. A is a nilpotent matrix
2. A is a diagonalizable matrix
3. the rank of A is 2
4. The Jordan canonical form of A is
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

113. For every 4×4 real symmetric non-singular matrix A, there exists a positive integer p such that
1. pI + A is positive definite
2. A^p is positive definite
3. A^{-p} is positive definite
4. exp(pA) – I is positive definite

114. Let A be an $m\times n$ matrix of rank m with $n > m$. If for some non-zero real number α, we have $x^t AA^t x=\alpha\, x^t x$, for all $x\in\mathbb{R}^m$ then A^tA has
1. exactly two distinct eigenvalues
2. 0 as an eigenvalue with multiplicity n-m
3. α as a non – zero eigenvalue
4. exactly two non-zero distinct eigenvalues

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PART – B

115. Let $\mathbb{R}^n$, $n \geq 2$, be equipped with standard inner product. Let $\{v_1, v_2,..., v_n\}$ be n column vectors forming an orthonormal basis of $\mathbb{R}^n$. Let A be the $n \times n$ matrix formed by the column vectors $v_1, ..., v_n$. Then
1. $A = A^{-1}$
2. $A = A^T$
3. $A^{-1} = A^T$
4. Det(A) = 1

116. Let A be a $(m\times n)$ matrix and B be a $(n\times m)$ matrix over real numbers with $m < n$. Then
1. AB is always nonsingular
2. AB is always singular
3. BA is always nonsingular
4. BA is always singular

117. If A is a (2×2) matrix over $\mathbb{R}$ with Det(A+I) =1+Det(A), then we can conclude that
1. Det(A) = 0
2. A=0
3. Tr(A) = 0
4. A is nonsingular

118. The system of equations:
$$-1 \cdot x+2\cdot x^2+3\cdot xy+0\cdot y=6$$
$$2 \cdot x+1\cdot x^2+3\cdot xy+1\cdot y=5$$
$$3 \cdot x-1\cdot x^2+0\cdot xy+1\cdot y=7$$
1. has solutions in rational numbers
2. has solutions in real numbers
3. has solutions in complex numbers
4. has no solution

119. The trace of the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}^{20}$ is
1. 7^{20}
2. $2^{20} + 3^{20}$
3. $2\cdot 2^{20} + 3^{20}$
4. $2^{20} + 3^{20} +1$

PART – C

120. Let $A=\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ and define for x, y, $z \in\mathbb{R}$ $Q(x,y,z)=(x\ y\ z)A\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

Which of the following statements are true?
1. The matrix of second order partial derivatives of the quadratic form Q is 2A.
2. The rank of the quadratic form Q is 2
3. The signature of the quadratic form Q is (+ + 0)
4. The quadratic form Q takes the value 0 for some non-zero vector (x, y, z)

121. Let $M_n(\mathbb{R})$ denote the space of all $n \times n$ real matrices identified with the Euclidean space $\mathbb{R}^{n^2}$. Fix a column vector $x \neq 0$ in $\mathbb{R}^n$. Define $f:M_n(\mathbb{R}) \to \mathbb{R}$ by $f(A) = \langle A^2x, x\rangle$. Then
1. f is linear
2. f is differentiable
3. f is continuous but not differentiable
4. f is unbounded

122. Let V denote the vector space of all sequences **a** = $(a_1, a_2, \ldots)$ of real numbers such that $\sum 2^n|a_n|$ converges.

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Define $||.|| : V \to \mathbb{R}$ by $||\mathbf{a}|| = \sum 2^n |a_n|$.
Which of the following are true?
1. V contains only the sequence (0, 0, …)
2. V is finite dimensional
3. V has a countable linear basis
4. V is a complete normed space

123. Let V be a vector space over $\mathbb{C}$ with dimension n. Let $T : V \to V$ be a linear transformation with only 1 as eigenvalue. Then which of the following must be true?
1. $T - I = 0$
2. $(T - I)^{n-1} = 0$
3. $(T - I)^n = 0$
4. $(T - I)^{2n} = 0$

124. If A is a (5×5) matrix and the dimension of the solution space of $A\mathbf{x} = 0$ is at least two, then
1. Rank $(A^2) \le 3$
2. Rank $(A^2) \ge 3$
3. Rank $(A^2) = 3$
4. Det $(A^2) = 0$

125 Let $A \in M_3(\mathbb{R})$ be such that $A^8 = I_{3\times3}$. Then
1. minimal polynomial of A can only be of degree 2
2. minimal polynomial of A can only be of degree 3
3. either $A = I_{3\times3}$ or $A = -I_{3\times3}$
4. there are uncountably many A satisfying the above

126. Let A be an $n \times n$ matrix (with $n > 1$) satisfying $A^2 - 7A + 12I_{n\times n} = O_{n\times n}$, where $I_{n\times n}$ and $O_{n\times n}$ denote the identity matrix and zero matrix of order n respectively. Then which of the following statements are true?
1. A is invertible
2. $t^2 - 7t + 12n = 0$ where $t = \mathrm{Tr}(A)$
3. $d^2 - 7d + 12 = 0$ where $d = \mathrm{Det}(A)$
4. $\lambda^2 - 7\lambda + 12 = 0$ where λ is an eigenvalue of A

127. Let A be a (6×6) matrix over $\mathbb{R}$ with characteristic polynomial $= (x - 3)^2 (x - 2)^4$ and minimal polynomial $= (x - 3)(x - 2)^2$. Then Jordan canonical form of A can be

1. $\begin{pmatrix} 3&0&0&0&0&0\\ 0&3&0&0&0&0\\ 0&0&2&1&0&0\\ 0&0&0&2&1&0\\ 0&0&0&0&2&1\\ 0&0&0&0&0&2 \end{pmatrix}$

2. $\begin{pmatrix} 3&0&0&0&0&0\\ 0&3&0&0&0&0\\ 0&0&2&1&0&0\\ 0&0&0&2&0&0\\ 0&0&0&0&2&0\\ 0&0&0&0&0&2 \end{pmatrix}$

3. $\begin{pmatrix} 3&0&0&0&0&0\\ 0&3&0&0&0&0\\ 0&0&2&1&0&0\\ 0&0&0&2&0&0\\ 0&0&0&0&2&1\\ 0&0&0&0&0&2 \end{pmatrix}$

4. $\begin{pmatrix} 3&1&0&0&0&0\\ 0&3&0&0&0&0\\ 0&0&2&1&0&0\\ 0&0&0&2&0&0\\ 0&0&0&0&2&1\\ 0&0&0&0&0&2 \end{pmatrix}$

128. Let V be an inner product space and S be a subset of V. Let $\bar{S}$ denote the closure of S in V with respect to the topology induced by the metric given by the inner product. Which of the following statements are true?
1. $S = (S^\perp)^\perp$
2. $\bar{S} = (S^\perp)^\perp$
3. $\overline{span(S)} = (S^\perp)^\perp$
4. $S^\perp = ((S^\perp)^\perp)^\perp$

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PART – B

129. Consider the subspaces W_1 and W_2 of $\mathbb{R}^3$ given by $W_1 = \{(x,y,z) \in \mathbb{R}^3 : x + y + z = 0\}$ and $W_2 = \{(x,y,z) \in \mathbb{R}^3 : x - y + z = 0\}$. If W is a subspace of $\mathbb{R}^3$ such that
(i) $W \cap W_2 =$ span $\{(0,1,1)\}$
(ii) $W \cap W_1$ is orthogonal to $W \cap W_2$ with respect to the usual inner product of $\mathbb{R}^3$, then
1. $W = span\ \{(0,1,-1),(0,1,1)\}$
2. $W = span\ \{(1,0,-1),(0,1,-1)\}$

3. $W = span\ \{(1,0,-1),(0,1,1)\}$
4. $W = span\ \{(1,0,-1),(1,0,1)\}$

130. Let $C = \left[\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}2\\1\end{pmatrix}\right]$ be a basis of $\mathbb{R}^2$ and T: $\mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x+y\\x-2y\end{pmatrix}$. If T[C] represents the matrix of T with respect to the basis C, then which among the following is true?

1. $T[C] = \begin{bmatrix}-3 & -2\\3 & 1\end{bmatrix}$
2. $T[C] = \begin{bmatrix}3 & -2\\-3 & 1\end{bmatrix}$
3. $T[C] = \begin{bmatrix}-3 & -1\\3 & 2\end{bmatrix}$
4. $T[C] = \begin{bmatrix}3 & -1\\-3 & 2\end{bmatrix}$

131. Let $W_1 = \{(u,v,w,x) \in \mathbb{R}^4 \mid u+v+w=0,\ 2v+x=0,\ 2u+2w-x=0\}$ and
$W_2 = \{(u,v,w,x) \in \mathbb{R}^4 \mid u+w+x=0,\ u+w-2x=0,\ v-x=0\}$. Then which of the following is true?
1. dim $(W_1) = 1$
2. dim $(W_2) = 2$
3. dim $(W_1 \cap W_2) = 1$
4. dim $(W_1 + W_2) = 3$

132. Let A be an n×n complex matrix. Assume that A is self-adjoint and let B denotes the inverse of $(A+iI)$. Then all eigenvalues of $(A-iI_n)$ B are
1. purely imaginary
2. of modulus one
3. real
4. of modulus less than one

133. Let $\{u_1,u_2,\ldots,u_n\}$ be an orthonormal basis of $\mathbb{C}^n$ as column vectors. Let $M=(u_1,\ldots,u_k)$, $N=(u_{k+1},\ldots,u_n)$ and P be the diagonal k×k matrix with diagonal entries $\alpha_1,\alpha_2,\ldots,\alpha_k \in \mathbb{R}$. Then which of the following is true?
1. Rank (MPM*)=k, whenever $\alpha_i \neq \alpha_j\ 1 \le i, j \le k$.
2. Trace (MPM*) = $\sum_{i=1}^{k} \alpha_i$
3. Rank (M*N)=min (k,n-k)
4. Rank (MM*+NN*)<n

134. Let B: $\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ be the function B(a,b) = ab. Which of the following is true?
1. B is a linear transformation
2. B is a positive definite bilinear form
3. B is symmetric but not positive definite
4. B is neither linear nor bilinear

PART – C

135. Let T : $\mathbb{R}^n \to \mathbb{R}^n$ be a linear map that satisfies $T^2 = T - I_n$. Then which of the following are true?
1. T is invertible
2. $T - I_n$ is not invertible
3. T has a real eigen value
4. $T^3 = -I_n$

136. Let $M = \begin{bmatrix}2 & 0 & 3 & 2 & 0 & -2\\0 & 1 & 0 & -1 & 3 & 4\\0 & 0 & 1 & 0 & 4 & 4\\1 & 1 & 1 & 0 & 1 & 1\end{bmatrix}$, $b_1 = \begin{bmatrix}5\\1\\1\\4\end{bmatrix}$ and $b_2 = \begin{bmatrix}5\\1\\3\\3\end{bmatrix}$. Then which of the following are true?
1. both systems $MX = b_1$ and $MX = b_2$ are inconsistent
2. both systems $MX = b_1$ and $MX = b_2$ are consistent
3. the system $MX = b_1 - b_2$ is consistent
4. the systems $MX = b_1 - b_2$ is inconsistent

137. Let $M = \begin{bmatrix}1 & -1 & 1\\2 & 1 & 4\\-2 & 1 & -4\end{bmatrix}$. Given that 1 is an eigen value of M, then which among the following are correct?
1. The minimal polynomial of M is $(X-1)(X+4)$
2. The minimal polynomial of M is $(X-1)^2(X+4)$
3. M is not diagonalizable
4. $M^{-1} = \frac{1}{4}(M+3I)$

138. Let A be a real matrix with characteristic polynomial $(X-1)^3$. Pick the correct statements from below:
1. A is necessarily diagonalizable
2. If the minimal polynomial of A is $(X-1)^3$, then A is diagonalizable
3. Characteristic polynomial of A^2 is $(X-1)^3$
4. If A has exactly two Jordan blocks, then $(A-I)^2$ is diagonalizable

139. Let P_3 be the vector space of polynomials with real coefficients and of degree at most 3. Consider the linear map $T : P_3 \to P_3$ defined by $T(p(x)) = p(x + 1) + p(x - 1))$. Which of the following properties does the matrix of T (with respect to the standard basis $B = \{1, x, x^2, x^3\}$ of P_3) satisfy?
1. det T = 0
2. $(T - 2I)^4 = 0$ but $(T - 2I)^3 \neq 0$
3. $(T - 2I)^3 = 0$ but $(T - 2I)^2 \neq 0$
4. 2 is an eigen value with multiplicity 4

140. Let M be an $n \times n$ Hermitian matrix of rank k, $k \neq n$. If $\lambda \neq 0$ is an eigen value of M with corresponding unit column vector u, with $Mu = \lambda u$, then which of the following are true?
1. rank $(M - \lambda uu^*) = k - 1$
2. rank $(M - \lambda uu^*) = k$
3. rank $(M - \lambda uu^*) = k + 1$
4. $(M - \lambda uu^*)^n = M^n - \lambda^n uu^*$

141. Define a real valued function B on $\mathbb{R}^2 \times \mathbb{R}^2$ as follows. If $u = (x_1, x_2)$, $w = (y_1, y_2)$ belong to $\mathbb{R}^2$ define $B(u, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$. Let $v_0 = (1, 0)$ and let $W = \{v \in \mathbb{R}^2 : B(v_0, v) = 0\}$. Then W
1. is not a subspace of $\mathbb{R}^2$
2. equals {(0, 0)}
3. is the y axis
4. is the line passing through (0, 0) and (1, 1)

142. Consider the Quadratic forms
$Q_1(x, y) = xy$
$Q_2(x, y) = x^2 + 2xy + y^2$
$Q_3(x, y) = x^2 + 3xy + 2y^2$ on $\mathbb{R}^2$. Choose the correct statements from below:
1. Q_1 and Q_2 are equivalent
2. Q_1 and Q_3 are equivalent
3. Q_2 and Q_3 are equivalent
4. all are equivalent

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PART – B

143. Consider the vector space P_n of real polynomials in x of degree less than or equal to n. Define $T : P_2 \to P_3$ by $(Tf)(x) = \int_0^x f(t)\,dt + f'(x)$. Then the matrix representation of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, x^2, x^3\}$ is

1. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \frac{1}{2} & 0 \\ 0 & 2 & 0 & \frac{1}{3} \end{pmatrix}$
2. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$
3. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{3} \end{pmatrix}$
4. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \frac{1}{2} \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

144. Let PA (x) denote the characteristic polynomial of a matrix A. Then for which of the following matrices, $P_A(x) - P_{A^{-1}}(x)$ is a constant?
1. $\begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix}$
2. $\begin{pmatrix} 4 & 3 \\ 2 & 3 \end{pmatrix}$
3. $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$
4. $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$

145. Which of the following matrices is not diagonalizable over $\mathbb{R}$?
1. $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$
2. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
3. $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$
4. $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$

146. What is the rank of the following matrix?
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$
1. 2
2. 3
3. 4
4. 5

147. Let V denote the vector space of real valued continuous functions on the closed interval [0, 1]. Let W be the subspace of V spanned by {sin(x), cos(x), tan(x)}. Then the dimension of W over $\mathbb{R}$ is
1. 1
2. 2
3. 3
4. infinite

148. Let V be the vector space of polynomials in the variable t of degree at most 2 over $\mathbb{R}$. An inner product on V is defined by

$$\langle f,g\rangle=\int_0^1 f(t)\,g(t)\,dt$$

for f, g $\in$ V. Let W = span $\{1-t^2, 1+t^2\}$ and $W^\perp$ be the orthogonal complement of W in V. Which of the following conditions is satisfied for all h $\in W^\perp$?
1. h is an even function, i.e. h(t) = h(-t)
2. h is an odd function, i.e. h(t) = -h(-t)
3. h(t) = 0 has a real solution
4. h(0) = 0

PART – C

149. Let $L(\mathbb{R}^n)$ be the space of $\mathbb{R}$-linear maps from $\mathbb{R}^n$ to $\mathbb{R}^n$. If Ker (T) denotes the kernel (null space) of T then which of the following are true?
1. There exists T $\in$ L $(\mathbb{R}^5)$ \ {0} such that Range (T) = Ker (T)
2. There does not exist T $\in$ L $(\mathbb{R}^5)$ \ {0} such that Range (T) = Ker (T)
3. There exists T $\in$ L $(\mathbb{R}^6)$ \ {0} such that Range (T) = Ker (T)
4. There does not exist T $\in$ L $(\mathbb{R}^6)$ \ {0} such that Range (T) = Ker (T)

150. Let V be a finite dimensional vector space over $\mathbb{R}$ and T : V $\to$ V be a linear map. Can you always write $T = T_2 \circ T_1$ for some linear maps T_1 : V $\to$ W, T_2 : W $\to$ V, where W is some finite dimensional vector space and such that
1. both T_1 and T_2 are onto
2. both T_1 and T_2 are one to one
3. T_1 is onto, T_2 is one to one
4. T_1 is one to one, T_2 is onto

151. Let A = $((a_{ij}))$ be a 3 $\times$ 3 complex matrix. Identify the correct statements
1. $\det(((-1)^{i+j}a_{ij}))=\det A$
2. $\det(((-1)^{i+j}a_{ij}))=-\det A$
3. $\det(((\sqrt{-1})^{i+j}a_{ij}))=\det A$
4. $\det(((\sqrt{-1})^{i+j}a_{ij}))=-\det A$

152. Let $p(x) = a_0 + a_1x + \ldots + a_nx^n$ be a non-constant polynomial of degree n $\geq$ 1. Consider the polynomial

$$q(x)=\int_0^x p(t)\,dt,\ r(x)=\frac{d}{dx}p(x).$$

Let V denote the real vector space of all polynomials in x. Then which of the following are true?
1. q and r are linearly independent in V
2. q and r are linearly dependent in V
3. x^n belongs to the linear span of q and r
4. x^{n+1} belongs to the linear span of q and r

153. Let $M_n(\mathbb{R})$ be the ring of n $\times$ n matrices over $\mathbb{R}$. Which of the following are true for every n $\geq$ 2?
1. there exist matrices A, B $\in M_n(\mathbb{R})$ such that AB – BA = I_n, where I_n denotes the identity n $\times$ n matrix.
2. if A, B $\in M_n(\mathbb{R})$ and AB = BA, then A is diagonalizable over $\mathbb{R}$ if and only if B is diagonalizable over $\mathbb{R}$
3. if A, B $\in M_n(\mathbb{R})$, then AB and BA have same minimal polynomial
4. if A, B $\in M_n(\mathbb{R})$, then AB and BA have the same eigen values in $\mathbb{R}$

154. Consider a matrix A = $(a_{ij})_{5\times5}$, 1 $\leq$ i, j $\leq$ 5 such that $a_{ij}=\frac{1}{n_i+n_j+1}$, where $n_i, n_j \in \mathbb{N}$. Then in which of the following cases A is a positive definite matrix?
1. n_i = i for all i = 1, 2, 3, 4, 5
2. $n_1 < n_2 < \ldots < n_5$
3. $n_1 = n_2 = \ldots = n_5$
4. $n_1 > n_2 > \ldots > n_5$

155. Let $\langle\ ,\ \rangle : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ denote the standard inner product on $\mathbb{R}^n$. For a non zero w $\in \mathbb{R}^n$, define $T_w : \mathbb{R}^n \to \mathbb{R}^n$ by

$T_w(v)=v-\frac{2\langle v,w\rangle}{\langle w,w\rangle}w$, for v $\in \mathbb{R}^n$. Which of the following are true?
1. det (T_w) = 1
2. $\langle T_w(v_1), T_w(v_2)\rangle = \langle v_1, v_2\rangle\ \forall\ v_1, v_2 \in \mathbb{R}^n$
3. $T_w = T_w^{-1}$
4. $T_{2w} = 2T_w$

156. Consider the matrix $A=\begin{pmatrix}0&1\\1&0\end{pmatrix}$ over the field $\mathbb{Q}$ of rationals. Which of the following matrices are of the form P^tAP for a suitable

2×2 invertible matrix P over $\mathbb{Q}$? Here P^t denotes the transpose of P.

1. $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$
2. $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
4. $\begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix}$

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PART – B

157. Let $A = \begin{pmatrix} 2 & 0 & 5 \\ 1 & 2 & 3 \\ -1 & 5 & 1 \end{pmatrix}$. The system of linear equations AX = Y has a solution

1. only for $Y = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$, $x \in \mathbb{R}$
2. only for $Y = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$, $y \in \mathbb{R}$
3. only for $Y = \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}$, $y, z \in \mathbb{R}$
4. for all $Y \in \mathbb{R}^3$

158. Let V be a vector space of dimension 3 over $\mathbb{R}$. Let $T : V \to V$ be a linear transformation, given by the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -4 & 3 \\ -2 & 5 & -3 \end{pmatrix}$ with respect to an ordered basis (v_1, v_2, v_3) of V. Then which of the following statements is true?

1. $T(v_3) = 0$
2. $T(v_1 + v_2) = 0$
3. $T(v_1 + v_2 + v_3) = 0$
4. $T(v_1 + v_3) = T(v_2)$

159. Let $M_4(\mathbb{R})$ be the space of all (4×4) matrices over $\mathbb{R}$. Let

$W = \left\{ (a_{ij}) \in M_4(\mathbb{R}) \mid \right\}$
$\sum_{i+j=k} a_{ij} = 0, \text{ for } k = 2,3,4,5,6,7,8$

Then dim(W) is

1. 7
2. 8
3. 9
4. 10

160. For $t \in \mathbb{R}$, define

$$M(t) = \begin{pmatrix} 1 & t & 0 \\ 1 & 1 & t^2 \\ 0 & 1 & 1 \end{pmatrix}.$$

Then which of the following statements is true?

1. det M(t) is a polynomial function of degree 3 in t
2. det M(t) = 0 for all $t \in \mathbb{R}$
3. det M(t) is zero for infinitely many $t \in \mathbb{R}$
4. det M(t) is zero for exactly two $t \in \mathbb{R}$

161. For a quadratic form in 3 variables over $\mathbb{R}$, let r be the rank and s be the signature. The number of possible pairs (r, s) is

1. 13
2. 9
3. 10
4. 16

PART – C

162. Let $A \in M_3(\mathbb{R})$ and let $X = \{C \in GL_3(\mathbb{R}) \mid CAC^{-1}$ is triangular$\}$. Then

1. $X \neq \varnothing$
2. If $X = \varnothing$, then A is not diagonalizable over $\mathbb{C}$
3. If $X = \varnothing$. Then A is diagonalizable over $\mathbb{C}$
4. If $X = \varnothing$, then A has no real eigenvalue

163. Which of the following statements regarding quadratic forms in 3 variables are true?

1. Any two quadratic forms of rank 3 are isomorphic over $\mathbb{R}$
2. Any two quadratic forms of rank 3 are isomorphic over $\mathbb{C}$
3. There are exactly three non zero quadratic forms of rank ≤ 3 upto isomorphism over $\mathbb{C}$
4. There are exactly three non zero quadratic forms of rank 2 upto isomorphism over $\mathbb{R}$ and $\mathbb{C}$

164. Let $T : \mathbb{C}^n \to \mathbb{C}^n$ be a linear transformation, $n \geq 2$. Suppose 1 is the only eigenvalue of T. Which of the following statements are true?

1. $T^k \neq I$ for any $k \in \mathbb{N}$
2. $(T - I)^{n-1} = 0$

3. $(T - I)^n = 0$
4. $(T - I)^{n+1} = 0$

165. Let X be a finite dimensional inner product space over $\mathbb{C}$. Let $T : X \to X$ be any linear transformation. Then which of the following statements are true?
1. T is unitary $\Rightarrow$ T is self adjoint
2. T is self adjoint $\Rightarrow$ T is normal
3. T is unitary $\Rightarrow$ T is normal
4. T is normal $\Rightarrow$ T is unitary

166. Let $n \geq 1$ and $\alpha, \beta \in \mathbb{R}$ with $\alpha \neq \beta$. Suppose $A_n(\alpha, \beta) = [a_{ij}]$ is an $n \times n$ matrix such that $a_{ii} = \alpha$ and $a_{ij} = \beta$ for $i \neq j$, $1 \leq i, j \leq n$. Let D_n be the determinant of $A_n(\alpha, \beta)$. Which of the following statements are true?
1. $D_n = (\alpha - \beta)D_{n-1} + \beta$ for $n \geq 2$
2. $\dfrac{D_n}{(\alpha-\beta)^{n-1}} = \dfrac{D_{n-1}}{(\alpha-\beta)^{n-2}} + \beta$ for $n \geq 2$
3. $D_n = (\alpha + (n-1)\beta)^{n-1}(\alpha - \beta)$ for $n \geq 2$
4. $D_n = (\alpha + (n-1)\beta)(\alpha - \beta)^{n-1}$ for $n \geq 2$

167. Which of the following statements are true?
1. Any two quadratic forms of same rank in n-variables over $\mathbb{R}$ are isomorphic
2. Any two quadratic forms of same rank in n-variables over $\mathbb{C}$ are isomorphic
3. Any two quadratic forms in n-variables are isomorphic over $\mathbb{C}$
4. A quadratic form in 4 variables may be isomorphic to a quadratic from in 10 variables

168. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation with characteristic polynomial $(x - 2)^4$ and minimal polynomial $(x - 2)^2$. Jordan canonical form of T can be

1. $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$ 2. $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

3. $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ 4. $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

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PART – B

169. Let A be an $n \times n$ matrix such that the set of all its non-zero eigenvalues has exactly r elements. Which of the following statements is true?
1. rank $A \leq r$
2. If $r = 0$, then rank $A < n - 1$
3. rank $A \geq r$
4. A^2 has r distinct non zero eigenvalues

170. Let A and B be 2×2 matrices. Then which of the following is true?
1. $\det(A + B) + \det(A - B) = \det A + \det B$
2. $\det(A + B) + \det(A - B) = 2\det A - 2\det B$
3. $\det(A + B) + \det(A - B) = 2\det A + 2\det B$
4. $\det(A + B) - \det(A - B) = 2\det A - 2\det B$

171. If $A = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$, then A^{20} equals

1. $\begin{pmatrix} 41 & 40 \\ -40 & -39 \end{pmatrix}$ 2. $\begin{pmatrix} 41 & -40 \\ 40 & -39 \end{pmatrix}$

3. $\begin{pmatrix} 41 & -40 \\ -40 & -39 \end{pmatrix}$ 4. $\begin{pmatrix} 41 & 40 \\ 40 & -39 \end{pmatrix}$

172. Let A be a 2×2 real matrix with det A = 1 and trace A = 3. What is the value of trace A^2?
1. 2 2. 10
3. 9 4. 7

173. For $a, b \in \mathbb{R}$, let
$p(x, y) = a^2x_1y_1 + abx_2y_1 + abx_1y_2 + b^2x_2y_2$, $x = (x_1, x_2)$, $y = (y_1, y_2) \in \mathbb{R}^2$.
For what values of a and b does
$p : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ define an inner product?
1. $a > 0, b > 0$
2. $ab > 0$
3. $a = 0, b = 0$
4. For no values of a, b

174. Which of the following real quadratic forms on $\mathbb{R}^2$ is positive definite?
1. $Q(X, Y) = XY$
2. $Q(X, Y) = X^2 - XY + Y^2$
3. $Q(X, Y) = X^2 + 2XY + Y^2$
4. $Q(X, Y) = X^2 + XY$

PART – C

175. Let P be a square matrix such that $P^2 = P$. Which of the following statements are true?
1. Trace of P is an irrational number
2. Trace of P = rank of P
3. Trace of P is an integer
4. Trace of P is an imaginary complex number

176. Let A and B be $n \times n$ real matrices and let $C = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$.
Which of the following statements are true?
1. If λ is an eigenvalue of A + B then λ is an eigenvalue of C
2. If λ is an eigenvalue of A – B then λ is an eigenvalue of C
3. If λ is an eigenvlaue of A or B then λ is an eigenvalue of C
4. All eigenvalues of C are real

177. Let A be an $n \times n$ real matrix. Let b be an $n \times 1$ vector. Suppose Ax = b has no solution. Which of the following statements are true?
1. There exists an $n \times 1$ vector c such that Ax = c has a unique solution
2. There exist infinitely many vectors c such that Ax = c has no solution
3. If y is the first column of A then Ax = y has a unique solution
4. det A = 0

178. Let A be an $n \times n$ matrix such that the first 3 rows of A are linearly independent and the first 5 columns of A are linearly independent. Which of the following statements are true?
1. A has atleast 5 linearly independent rows
2. $3 \le$ rank $A \le 5$
3. rank $A \ge 5$
4. rank $A^2 \ge 5$

179. Let n be a positive integer and F be a non-empty proper subset of {1, 2, …, n}. Define $\langle x, y\rangle_F = \sum_{k\in F} x_k y_k$, $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n) \in \mathbb{R}^n$.
Let $T = \{x \in \mathbb{R}^n : \langle x, x\rangle_F = 0\}$. Which of the following statements are true?
For $y \in \mathbb{R}^n$, $y \neq 0$
1. $\inf_{x\in T} \langle x+y, x+y\rangle_F = \langle y, y\rangle_F$
2. $\sup_{x\in T} \langle x+y, x+y\rangle_F = \langle y, y\rangle_F$
3. $\inf_{x\in T} \langle x+y, x+y\rangle_F < \langle y, y\rangle_F$
4. $\sup_{x\in T} \langle x+y, x+y\rangle_F > \langle y, y\rangle_F$

180. Let $v \in \mathbb{R}^3$ be a non-zero vector. Define a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ by
$T(x) = x - 2\frac{x.v}{v.v}v$, where x . y denotes the standard inner product in $\mathbb{R}^3$. Which of the following statements are true?
1. The eigenvalues of T are +1, -1
2. The determinant of T is -1
3. The trace of T is +1
4. T is distance preserving

181. A quadratic form Q(x,y,z) over $\mathbb{R}$ represents 0 non trivially if there exists $(a,b,c) \in \mathbb{R}^3 \setminus \{(0,0,0)\}$ such that Q(a, b, c) = 0. Which of the following quadratic forms Q(x, y, z) over $\mathbb{R}$ represent 0 non trivially?
1. $Q(x, y, z) = xy + z^2$
2. $Q(x, y, z) = x^2 + 3y^2 - 2z^2$
3. $Q(x, y, z) = x^2 - xy + y^2 + z^2$
4. $Q(x, y, z) = x^2 + xy + z^2$

182. Let Q(x, y, z) be a real quadratic form. Which of the following statements are true?
1. $Q(x_1 + x_2, y, z) = Q(x_1, y, z) + Q(x_2, y, z)$ for all x_1, x_2, y, z
2. $Q(x_1 + x_2, y_1 + y_2, 0) + Q(x_1 - x_2, y_1 - y_2, 0) = 2Q(x_1, y_1, 0) + 2Q(x_2, y_2, 0)$ for all x_1, x_2, y_1, y_2
3. $Q(x_1 + x_2, y_1 + y_2, z_1 + z_2) = Q(x_1, y_1, z_1) + Q(x_2, y_2, z_2)$ for at least one choice of $x_1, x_2, y_1, y_2, z_1, z_2$
4. $2Q(x_1 + x_2, y_1 + y_2, 0) + 2Q(x_1 - x_2, y_1 - y_2, 0) = Q(x_1, y_1, 0) + Q(x_2, y_2, 0)$ for all x_1, x_2, y_1, y_2

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PART – B

183. Let A be a 4×4 matrix such that -1, 1, 1, -2 are its eigenvalues. If $B = A^4 - 5A^2 + 5I$, then trace (A + B) equals
(1) 0
(2) -12
(3) 3
(4) 9

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184. Let $M = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 2 & -1 \\ 1 & 1 & 3 \end{bmatrix}$. Given that 1 is an eigenvalue of M, which of the following statements is true?
(1) -2 is an eigenvalue of M
(2) 3 is an eigenvalue of M
(3) The eigen space of each eigen value has dimension 1
(4) M is diagonalizable

185. Let A and B be n × n matrices. Suppose the sum of the elements in any row of A is 2 and the sum of the elements in any column of B is 2. Which of the following matrices is necessarily singular?
(1) $I - \frac{1}{2}BA^T$ (2) $I - \frac{1}{2}AB$
(3) $I - \frac{1}{4}AB$ (4) $I - \frac{1}{4}BA^T$

186. Let V = {A ∈ $M_{3\times3}(\mathbb{R})$: $A^t + A \in \mathbb{R}$. I) where I is the identity matrix. Consider the quadratic form defined as q(A) = Trace$(A)^2$ – Trace (A^2). What is the signature of the quadratic form?
(1) (+ + + +) (2) (+ 0 0 0)
(3) (+ - - -) (4) (- - - 0)

187. Let n > 1 be a fixed natural number. Which of the following is an inner product on the vector space of n × n real symmetric matrices?
(1) ⟨A, B⟩ = (trace(A)) (trace(B))
(2) ⟨A, B⟩ = trace (AB)
(3) ⟨A, B⟩ = determinant (AB)
(4) ⟨A, B⟩ = trace (A) + trace (B)

188. Consider the two statements given below:
I. There exists a matrix N ∈$\mathbb{M}_4(\mathbb{R})$ such that {(1, 1, 1, -1), (1, -1, 1, 1)} is a basis of Row(N) and (1, 2, 1, 4) ∈ Null (N)
II. There exists a matrix M ∈$\mathbb{M}_4(\mathbb{R})$ such that $\{(1, 1, 1, 0)^T, (1, 0, 1, 1)^T\}$ is a basis of Col(M) and $(1, 1, 1, 1)^T, (1, 0, 1, 0)^T \in$ Null (M)

Which of the following statements is true?
(1) Statement I is False and Statement II is True
(2) Statement I is True and Statement II is False
(3) Both Statement I and Statement II are False
(4) Both Statement I and Statement II are True

PART – C

189. Let M ∈$\mathbb{M}_n(\mathbb{R})$ such that M ≠ 0 but M^2 = 0. Which of the following statements are true?
(1) If n is even then dim(Col)(M)) > dim(Null(M))
(2) If n is even then dim(Col(M)) ≤ dim(Null(M))
(3) If n is odd then dim(Col(M)) < dim(Null(M))
(4) If n is odd then dim(Col(M)) > dim(Null(M))

190. Consider the system
2x + ky = 2 – k
kx + 2y = k
ky + kz = k – 1
in three unknowns and one real parameter k. For which of the following values of k is the system of linear equations consistent?
(1) 1 (2) 2
(3) -1 (4) -2

191. Which of the following are inner products on $\mathbb{R}^2$?
(1) $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + 2x_1y_2 + 2x_2y_1 + x_2y_2$
(2) $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2$
(3) $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$
(4) $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 - \frac{1}{2}x_1y_2 + -\frac{1}{2}x_2y_1 + x_2y_2$

192. Let A be an m × n matrix such that the first r rows of A are linearly independent and the first s columns of A are linearly independent, where r < m and s < n. Which of the following statements are true?
(1) The rank of A is atleast max{r, s}
(2) The submatrix formed by the first r rows and the first s columns of A has rank min{r, s}
(3) If r < s, then there exists a row among rows r + 1, …, m which together with the first r rows form a linearly independent set

(4) If s < r, then there exists a column among columns s + 1, …, n which together with the first s columns form a linearly dependent set.

193. Let A be an $n \times n$ matrix. We say that A is diagonalizable if there exists a nonsingular matrix P such that PAP^{-1} is a diagonal matrix. Which of the following conditions imply that A is diagonalizable?
(1) There exists integer k such that $A^k = 1$
(2) There exists integer k such that A^k is nilpotent
(3) A^2 is diagonalizable
(4) A has n linearly independent eigenvectors

194. It is known that $X = X_0 \in M_2(\mathbb{Z})$ is a solution of $AX - XA = A$ for some

$$A \in \left\{\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right\}$$

Which of the following values are NOT possible for the determinant of X_0?
(1) $\det(X_0) = 0$
(2) $\det(X_0) = 2$
(3) $\det(X_0) = 6$
(4) $\det(X_0) = 10$

195. Let A be an $m \times m$ matrix with real entries and let x be an $m \times 1$ vector of unknowns. Now consider the two statements given below:

I: There exists non-zero vector $b_1 \in \mathbb{R}^m$ such that the linear system $Ax = b_1$ has NO solution

II: There exist non-zero vectors $b_2, b_3 \in \mathbb{R}^m$, with $b_2 \neq cb_3$ for any $c \in \mathbb{R}$, such that the linear systems $Ax = b_2$ and $Ax = b_3$ have solutions.

Which of the following statements are true?
(1) II is TRUE whenever A is singular
(2) I is TRUE whenever A is singular
(3) Both I and II can be TRUE simultaneously
(4) If m = 2, then at least one of I and II is FALSE

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PART – B

196. Let $A = (a_{i,j})$ be a real symmetric 3×3 matrix. Consider the quadratic form $Q(x_1, x_2, x_3) = x^tAx$ where $x = (x_1, x_2, x_3)^t$. Which of the following is true?
(1) If $Q(x_1, x_2, x_3)$ is positive definite, then $a_{i,j} > 0$ for all $i \neq j$.
(2) If $Q(x_1, x_2, x_3)$ is positive definite, then $a_{i,i} > 0$ for all i.
(3) If $a_{i,j} > 0$ for all $i \neq j$, then $Q(x_1, x_2, x_3)$ is positive definite.
(4) If $a_{i,i} > 0$ for all i, then $Q(x_1, x_2, x_3)$is positive definite.

197. Let $\mathbb{R}$ be the field of real numbers. Let V be the vector space of real polynomials of degree at most 1. Consider the bilinear form

$\langle , \rangle : V \times V \to \mathbb{R}$,

given by $\langle f, g\rangle = \int_0^1 f(x)\,g(x)\,dx.$

Which of the following is true?
(1) For all non-zero real numbers a, b, there exists a real number c such that the vectors ax + b, x + c ∈ V are orthogonal to each other.
(2) For all non-zero real numbers b, there are infinitely many real numbers c such that the vectors x + b, x + c ∈ V are orthogonal to each other.
(3) For all positive real numbers c, there exist infintely many real numbers a, b such that the vectors ax + b, x + c ∈ V are orthogonal to each other.
(4) For all non-zero real numbers b, there are infinitely many real numbers c such that the vectors b, x + c ∈ V are orthogonal to each other.

198. Suppose A is a real $n \times n$ matrix of rank r. Let V be the vector space of all real $n \times n$ matrices X such that AX = 0. What is the dimension of V?
(1) r
(2) nr
(3) n^2r
(4) $n^2 - nr$

199. Suppose A and B are similar real matrices, that is, there exists an invertible matrix S such that $A = SBS^{-1}$. Which of the following need not be true?
(1) Transpose of A is similar to the transpose of B
(2) The minimal polynomial of A is same as the minimal polynomial of B
(3) trace(A) = trace(B)
(4) The range of A is same as the range of B

200. Let A be an invertible 5×5 matrix over a field F. Suppose that characteristic polynomials of A and A^{-1} are the same. Which of the following is necessarily true?

(1) $\det(A)^2 = 1$ (2) $\det(A)^5 = 1$
(3) trace $(A)^2 = 1$ (4) trace $(A)^5 = 1$

PART – C

201. Let W be the space of $\mathbb{C}$-linear combinations of the following functions $f_1(z) = \sin z$, $f_2(z) = \cos z$, $f_3(z) = \sin(2z)$, $f_4(z) = \cos(2z)$
Let T be the linear operator on W given by complex differentiation. Which of the following statements are true?
(1) Dimension of W is 3
(2) The span of f_1 and f_2 is Jordan block of T
(3) T has two Jordan blocks
(4) T has four Jordan blocks

202. Let V be vector space of polynomials $f(X, Y) \in \mathbb{R}[X, Y]$ with (total) degree less than 3. Let $T : V \to V$ be the linear transformation given by $\frac{\partial}{\partial x}$. Which of the following statements are true?
(1) The nullity of T is atleast 3
(2) The rank of T is atleast 4
(3) The rank of T is atleast 3
(4) T is invertible

203. For a positive integer $n \geq 2$, let A be an $n \times n$ matrix with entries in $\mathbb{R}$ such that A^{n^2} has rank zero. Let 0_n denote the $n \times n$ matrix with all entries equal to 0. Which of the following statements are equivalent to the statement that A has n linearly indepednent eigenvectors?
(1) $A^n = 0_n$ (2) $A^{n^2} = 0_n$
(3) $A = 0_n$ (4) $A^2 = 0_n$

204. Let P_n be the vector space of real polynomials with degree at most n. Let $\langle\ ,\ \rangle$ be an inner product on P_n with respect to which $\left\{1, x, \frac{1}{2!}x^2, \ldots, \frac{1}{n!}x^n\right\}$ is an orthonormal basis of P_n.
Let $f = \sum_i \alpha_i x^i$, $g = \sum_i \beta_i x^i \in P_n$. Which of the following statements are true?
(1) $\langle f, g\rangle = \sum_i (i!)\alpha_i\beta_i$ defines one such inner product, but there is another such inner product.
(2) $\langle f, g\rangle = \sum_i (i!)\alpha_i\beta_i$.
(3) $\langle f, g\rangle = \sum_i (i!)^2\alpha_i\beta_i$ defines one such inner product, but there is another such inner product.
(4) $\langle f, g\rangle = \sum_i (i!)^2\alpha_i\beta_i$

205. Let U and V be the subspaces of $\mathbb{R}^3$ defined by

$$U = \left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid 2x + 3y + 4z = 0\right\}.$$

$$V = \left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + 2y + 5z = 0\right\}.$$

Which of the following statements are true?
(1) There exists an invertible linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(U) = V$.
(2) There does not exists an invertible linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(V) = U$.
(3) There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(U) \cap V \neq \{0\}$ and the characteristic polynomial of T is not the product of linear polynomials with real coefficients.
(4) There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(U) = V$ and the characteristic polynomial of T vanishes at 1.

206. Let A be an $n \times n$ matrix with entries in $\mathbb{R}$ that A and A^2 are of same rank. Consider linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(v) = A(v)$ for all $v \in \mathbb{R}^n$. Which of the following statements are true.
(1) The Kernels of T and $T \circ T$ are the same
(2) The Kernels of T and $T \circ T$ are of equal dimensions
(3) A must be invertible
(4) $I_n + A$ must be invertible, where I_n denotes at $n \times n$ identity matrix

207. On the complex vector space $\mathbb{C}^{100}$ with standard basis $\{e_1, e_2, \ldots, e_{100}\}$, consider the bilinear form $B(x, y) = \sum_i x_i y_i$, where x_i and y_i are the coefficients of e_i in x and y respectively. Which of the following statements are true?
(1) B is non-degenerate

(2) Restriction of B to all non-zero subspaces is non-degenerate
(3) There is a 51 dimensional subspace W of $\mathbb{C}^{100}$ such that the restriction B : $W \times W \to \mathbb{C}$ is the zero map
(4) There is a 49 dimensional subspace W of $\mathbb{C}^{100}$ such that the restriction B : $W \times W \to \mathbb{C}$ is the zero map

208. For a positive integer $n \geq 2$, let $M_n(\mathbb{R})$ denote the vector space of $n \times n$ matrices with entries in $\mathbb{R}$. Which of the following statements are true?

(1) The vector space $M_n(\mathbb{R})$ can be expressed as the union of a finite collection of its proper subspaces.
(2) Let A be an element of $M_n(\mathbb{R})$. Then, for any real number x and $\varepsilon > 0$, there exists a real number $y \in (x - \varepsilon, x + \varepsilon)$ such that $\det(yI + A) \neq 0$.
(3) Suppose A and B are two elements of $M_n(\mathbb{R})$ such that their characteristic polynomials are equal. If $A = C^2$ for some $C \in M_n(\mathbb{R})$, then $B = D^2$ for some $D \in M_n(\mathbb{R})$.
(4) For any subspace W of $M_n(\mathbb{R})$, there exists a linear transformation T : $M_n(\mathbb{R}) \to M_n(\mathbb{R})$ with W as its image.

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PART – B

209. Let T be a linear operator on $\mathbb{R}^3$. Let $f(X) \in \mathbb{R}[X]$ denote its characteristic polynomial. Consider the following statements.

(a) Suppose T is non-zero and 0 is an eigen value of T. If we write $f(X) = Xg(X)$ in $\mathbb{R}[X]$, then the linear operator g(T) is zero.
(b) Suppose 0 is an eigenvalue of T with atleast two linearly independent eigen vectors. If we write $f(X) = Xg(X)$ in $\mathbb{R}[X]$, then the linear operator g(T) is zero.

Which of the following is true?
(1) Both (a) and (b) are true.
(2) Both (a) and (b) are false.
(3) (a) is true and (b) is false.
(4) (a) is false and (b) is true.

210. Let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ denote vectors in $\mathbb{R}^n$ for a fixed $n \geq 2$. Which of the following defines an inner product on $\mathbb{R}^n$?

(1) $\langle x, y\rangle = \sum_{i,\, j=1}^{n} x_i y_j$
(2) $\langle x, y\rangle = \sum_{i,\, j=1}^{n} (x_i^2 + y_j^2)$
(3) $\langle x, y\rangle = \sum_{j=1}^{n} j^3 x_j y_j$
(4) $\langle x, y\rangle = \sum_{j=1}^{n} x_j y_{n-j+1}$

211. Consider the quadratic form Q(x, y, z) associated to the matrix

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

Let

$$S = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 \mid Q(a, b, c) = 0 \right\}$$

Which of the following statements is FALSE?
(1) The intersection of S with the xy-plane is a line
(2) The intersection of S with the xz-plane is an ellipse
(3) S is the union of two planes
(4) Q is a degenerate quadratic form

212. Let $l \geq 1$ be a positive integer. What is the dimension of the $\mathbb{R}$-vector space of all polynomials in two variables over $\mathbb{R}$ having a total degree of at most l?

(1) $l+1$
(2) $l(l-1)$
(3) $l(l+1)/2$
(4) $(l+1)(l+2)/2$

213. Let A be a 3×3 matrix with real entries. Which of the following assertions is FALSE?
(1) A must have a real eigenvalue
(2) If the determinant of A is 0, then 0 is an eigenvalue of A
(3) If the determinant of A is negative and 3 is an eigenvalue of A, the A must have three real eigenvalues
(4) If the determinant of A is positive and 3 is an eigenvalue of A, then A must have three real eigenvalues

214. Let A be a 3×3 real matrix whose characteristic polynomial p(T) is divisible by T^2. Which of the following statements is true?
(1) The eigenspace of A for the eigenvalue 0 is two-dimensional
(2) All the eigenvalues of A are real
(3) $A^3 = 0$
(4) A is diagonalizable

PART – C

215. Let V be the vector space of all polynomials in one variable of degree at most 10 with real coefficients. Let W_1 be the subspace of V consisting of polynomials of degree at most 5 and let W_2 be the subspace of V consisting of polynomials such that the sum of their coefficients is 0. Let W be the smallest subspace of V containing both W_1 and W_2. Which of the following statements are true?
(1) The dimension of W is at most 10
(2) W = V
(3) $W_1 \subset W_2$
(4) The dimension of $W_1 \cap W_2$ is at most 5

216. Let V be a finite dimensional real vector space and T_1, T_2 be two nilpotent operators on V. Let $W_1 = \{v \in V : T_1(v) = 0\}$ and $W_2 = \{v \in V : T_2(v) = 0\}$. Which of the following statements are FALSE?
(1) If T_1 and T_2 are similar, then W_1 and W_2 are isomorphic vector spaces
(2) If W_1 and W_2 are isomorphic vector spaces, then T_1 and T_2 have the same minimal polynomial
(3) If $W_1 = W_2 = V$, then T_1 and T_2 are similar
(4) If W_1 and W_2 are isomorphic, then T_1 and T_2 have the same characteristic polynomial

217. Consider the following quadratic forms over $\mathbb{R}$
(a) $6X^2 - 13XY + 6Y^2$,
(b) $X^2 - XY + 2Y^2$,
(c) $X^2 - XY - 2Y^2$.
Which of the following statements are true?
(1) Quadratic forms (a) and (b) are equivalent
(2) Quadratic forms (a) and (c) are equivalent
(3) Quadratic form (b) is positive definite
(4) Quadratic form (c) is positive definite

218. Suppose A is a 5×5 block diagonal real matrix with diagonal blocks

$$\begin{pmatrix} e & 1 \\ 0 & e \end{pmatrix}, \begin{pmatrix} e & 1 & 0 \\ 0 & e & 0 \\ 0 & 0 & e \end{pmatrix}.$$

Which of the following statements are true?
(1) The algebraic multiplicity of e in A is 5
(2) A is not diagonalizable
(3) The geometric multiplicity of e in A is 3
(4) The geometric multiplicity of e in A is 2

219. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation satisfying $T^3 - 3T^2 = -2I$, where $I : \mathbb{R}^3 \to \mathbb{R}^3$ is the identity transformation. Which of the following statements are true?
(1) $\mathbb{R}^3$ must admit a basis B_1 such that the matrix of T with respect to B_1 is symmetric.
(2) $\mathbb{R}^3$ must admit a basis B_2 such that the matrix of T with respect to B_2 is upper triangular.
(3) $\mathbb{R}^3$ must contain a non-zero vector v such that Tv = v.
(4) $\mathbb{R}^3$ must contain two linearly independent vectors v_1, v_2 such that $Tv_1 = v_1$ and $Tv_2 = v_2$.

220. Let B be a 3×5 matrix with entries from $\mathbb{Q}$. Assume that $\{v \in \mathbb{R}^5 \mid Bv = 0\}$ is a three-dimensional real vector space. Which of the following statements are true?
(1) $\{v \in \mathbb{Q}^5 \mid Bv = 0\}$ is a three-dimensional vector space over $\mathbb{Q}$.
(2) The linear transformation $T : \mathbb{Q}^3 \to \mathbb{Q}^5$ given by $T(v) = B^t v$ is injective
(3) The column span of B is two-dimensional
(4) The linear transformation $T : \mathbb{Q}^3 \to \mathbb{Q}^3$ given by $T(v) = BB^t v$ is injective

221. Let V be the real vector space of real polynomials in one variable with degree less than or equal to 10 (including the zero polynomial). Let $T : V \to V$ be the linear map defined by $T(p) = p'$, where p' denotes

the derivative of p. Which of the followng statements are correct?
(1) rank (T) = 10
(2) determinant (T) = 0
(3) trace (T) = 0
(4) All the eigenvalues of T are equal to 0

222. Let V be an inner product space and let v_1, v_2, $v_3 \in V$ be an orthogonal set of vectors. Which of the following statements are true?
(1) The vectors $v_1 + v_2 + 2v_3$, $v_2 + v_3$, $v_2 + 3v_3$ can be extended to a basis of V
(2) The vectors $v_1 + v_2 + 2v_3$, $v_2 + v_3$, $v_2 + 3v_3$ can be extended to an orthogonal basis of V
(3) The vectors $v_1 + v_2 + 2v_3$, $v_2 + v_3$, $2v_1 + v_2 + 3v_3$ can be extended to a basis of V
(4) The vectors $v_1 + v_2 + 2v_3$, $2v_1 + v_2 + v_3$, $2v_1 + v_2 + 3v_3$ can be extended to a basis of V

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PART – B

223. For $a \in \mathbb{R}$, let
$A_a = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & a \end{pmatrix}$. Which one of the following statements is true?
(1) A_a is positive definite for all $a < 3$.
(2) A_a is positive definite for all $a > 3$.
(3) A_a is positive definite for all $a \ge -2$.
(4) A_a is positive definite only for finitely many values of a.

224. We denote by I_n the $n \times n$ identity matrix. Which one of the following statements is true?
(1) If A is a real 3×2 matrix and B is a real 2×3 matrix such that $BA = I_2$, then $AB = I_3$.
(2) Let A be the real matrix $\begin{pmatrix} 3 & 3 \\ 1 & 2 \end{pmatrix}$. Then there is a matrix B with integer entries such that $AB = I_2$.
(3) Let A be the matrix $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ with entries in $\mathbb{Z}/6\mathbb{Z}$. Then there is a matrix B with entries in $\mathbb{Z}/6\mathbb{Z}$ such that $AB = I_2$.
(4) If A is a real non-zero 3×3 diagonal matrix, then there is a real matrix B such that $AB = I_3$.

225. Which one of the following statements is FALSE?
(1) The product of two 2×2 real matrices of rank 2 is of rank 2.
(2) The product of two 3×3 real matrices of rank 2 is of rank atmost 2.
(3) The product of two 3×3 real matrices of rank 2 is of rank atleast 2.
(4) The product of two 2×2 real matrices of rank 1 can be the zero matrix.

226. Let $A = (a_{i,j})$ be the $n \times n$ real matrix with $a_{i,j} = ij$ for all $1 \le i, j \le n$. If $n \ge 3$, which one of the following is an eigenvalue of A?
(1) 1 (2) n
(3) $n(n + 1)/2$ (4) $n(n + 1)(2n + 1)/6$

227. Let A be an $n \times n$ matrix with complex entries. If $n \ge 4$, which one of the following statements is true?
(1) A does not have any non-zero invariant subspace in $\mathbb{C}^n$.
(2) A has an invariant subspace in $\mathbb{C}^n$ of dimension $n – 3$.
(3) All eigenvalues of A are real numbers.
(4) A^2 does not have any invariant subspace in $\mathbb{C}^n$ of dimension $n – 1$.

228. Let (-, -) be a symmetric bilinear form on $\mathbb{R}^2$ such that there exists non-zero $v, w \in \mathbb{R}^2$ such that $(v, v) > 0 > (w, w)$ and $(v, w) = 0$. Let A be the 2×2 real symmetric matrix representing this bilinear form with respect to the standard basis. Which one of the following statements is true?
(1) $A^2 = 0$.
(2) rank A = 1.
(3) rank A = 0.
(4) There exists $u \in \mathbb{R}^2$, $u \ne 0$ such that $(u, u) = 0$.

PART – C

229. Consider the quadratic form $Q(x, y, z) = x^2 + xy + y^2 + xz + yz + z^2$. Which of the following statements are true?

(1) There exists a non-zero $u \in \mathbb{Q}^3$ such that $Q(u) = 0$.

(2) There exists a non-zero $u \in \mathbb{R}^3$ such that $Q(u) = 0$.

(3) There exist a non-zero $u \in \mathbb{C}^3$ such that $Q(u) = 0$.

(4) The real symmetric 3×3 matrix A which satisfies

$$Q(x,y,z) = [x\ y\ z]\ A \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ for all } x,$$

$y, z \in \mathbb{R}$ is invertible.

230. Let $\mathbb{F}$ be a finite field and V be a finite dimensional non-zero $\mathbb{F}$-vector space. Which of the following can NEVER be true?

(1) V is the union of 2 proper subspaces.
(2) V is the union of 3 proper subspaces.
(3) V has a unique basis.
(4) V has precisely two bases.

231. Suppose a 7×7 block diagonal complex matrix A has blocks

(0), (1), $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, and

$\begin{pmatrix} 2\pi i & 1 & 0 \\ 0 & 2\pi i & 0 \\ 0 & 0 & 2\pi i \end{pmatrix}$ along the diagonal.

Which of the following statements are true?

(1) The characteristic polynomial of A is $x^3(x - 1)(x - 2\pi i)^3$.
(2) The minimal polynomial of A is $x^2(x - 1)(x - 2\pi i)^3$.
(3) The dimensions of the eigenspaces for $0, 1, 2\pi i$ are 2, 1, 3 respectively.
(4) The dimensions of the eigenspaces for $0, 1, 2\pi i$ are 2, 1, 2 respectively.

232. Let $T : \mathbb{R}^5 \to \mathbb{R}^5$ be a $\mathbb{R}$-linear transformation. Suppose that (1, -1,2,4,0), (4,6,1,6,0) and (5,5,3,9,0) span the null space of T. Which of the following statements are true?

(1) The rank of T is equal to 2.

(2) Suppose that for every vector $v \in \mathbb{R}^5$, there exists n such that $T^n v = 0$. Then T^2 must be zero.

(3) Suppose that for every vector $v \in \mathbb{R}^5$, there exists n such that $T^n v = 0$. Then T^3 must be zero.

(4) (-2, -8, 3, 2, 0) is contained in the null space of T.

233. Let X, Y be two $n \times n$ real matrices such that $XY = X^2 + X + I$.
Which of the following statements are necessarily true?

(1) X is invertible (2) X + I is invertible
(3) XY = YX (4) Y is invertible

234. Consider $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Suppose $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = aA + bI$ for some a, $b \in \mathbb{Z}$. Which of the following statements are true?

(1) a + b > 8 (2) a + b < 7
(3) a + b is divisible by 2 (4) a > b

235. Let A be an $n \times n$ real symmetric matrix. Which of the following statements are necessarily true?

(1) A is diagonalizable.
(2) If $A^k = I$ for some positive integer k, then $A^2 = I$.
(3) If $A^k = 0$ for some positive integer k, then $A^2 = 0$.
(4) All eigenvalues of A are real.

236. Let A be a real diagonal matrix with characteristic polynomial $\lambda^3 - 2\lambda^2 - \lambda + 2$. Define a bilinear form $\langle v, w\rangle = v^t A w$ on $\mathbb{R}^3$. Which of the following statements are true?

(1) A is positive definite.
(2) A^2 is positive definite.
(3) There exists a non-zero $v \in \mathbb{R}^3$ such that $\langle v, v\rangle = 0$.
(4) rank A = 2.

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PART – B

237. Let V be the real vector space of 2×2 matrices with entires in $\mathbb{R}$. Let $T : V \to V$ denote the linear transformation defined by $T(B) = AB$ for all $B \in V$, where

$A=\begin{pmatrix}2 & 0\\ 0 & 1\end{pmatrix}$. What is the characteristic polynomial of T?
(1) $(x-2)(x-1)$ (2) $x^2(x-2)(x-1)$
(3) $(x-2)^2(x-1)^2$ (4) $(x^2-2)(x^2-1)$

238. Let $A : \mathbb{R}^m \to \mathbb{R}^n$ be a non-zero linear transformation. Which of the following statements is true?
(1) If A is one to one but not onto, then $m > n$
(2) If A is onto but not one to one, then $m < n$
(3) If A is bijective, then $m = n$
(4) If A is one to one, then $m = n$

239. Let $\begin{pmatrix}2 & a\\ b & c\end{pmatrix}$ be a 2×2 real matrix for which 6 is an eigenvalue. Which of the following statements is necessarily true?
(1) $24 - ab = 4c$ (2) $a + b = 8$
(3) $c = 6$ (4) $ab = 0$

240. Let $A=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&1&1\\0&0&1&1\end{pmatrix}$, and consider the symmetric bilinear form on $\mathbb{R}^4$ given by $\langle v, w\rangle = v^tAw$, for $v, w \in \mathbb{R}^4$. Which of the following statements is true?
(1) A is invertible
(2) There exist non-zero vectors v, w such that $\langle v, w\rangle = 0$
(3) $\langle u, v\rangle \neq \langle u, w\rangle$ for all non-zeor vectors u, v, w with $v \neq w$
(4) Every eigenvalue of A^2 is positive

241. For a quadratic form $f(x, y, z) \in \mathbb{R}[x, y, z]$, we say that $(a, b, c) \in \mathbb{R}^3$ is a zero of f if $f(a, b, c) = 0$. Which of the following quadratic forms has at least one zero different from (0, 0, 0)?
(1) $x^2 + 2y^2 + 3z^2$
(2) $x^2 + 2y^2 + 3z^2 - 2xy$
(3) $x^2 + 2y^2 + 3z^2 - 2xy - 2yz$
(4) $x^2 + 2y^2 - 3z^2$

242. Let A be a 10×10 real matrix. Assume that the rank of A is 7. Which of the following statements is necessarily true?
(1) There exists a vector $v \in \mathbb{R}^{10}$ such that $Av \neq 0$ and $A^2v = 0$
(2) There exists a vector $v \in \mathbb{R}^{10}$ such that $A^2v \neq 0$
(3) A must have a non-zero eigenvalue
(4) $A^7 = 0$

PART – C

243. Let $V (\neq \{0\})$ be a finite dimensional vector space over $\mathbb{R}$ and $T : V \to V$ be a linear operator. Suppose that the kernel of T equals the image of T. Which of the following statements are necessarily true?
(1) The dimension of V is even
(2) The trace of T is zero
(3) The minimal polynomial of T cannot have two distinct roots
(4) The minimal polynomial of T is equal to its characteristic polynomial

244. Consider $\mathbb{R}$ and $\mathbb{Q}[x]$ as vector spaces over $\mathbb{Q}$. Which of the followig statements are true?
(1) There exists an injective $\mathbb{Q}$-linear transformation $T : \mathbb{R} \to \mathbb{Q}[x]$
(2) There exists an injective $\mathbb{Q}$-linear transformation $T : \mathbb{Q}[x] \to \mathbb{R}$
(3) The $\mathbb{Q}$-vector spaces $\mathbb{Q}[x]$ and $\mathbb{R}$ are isomorphic
(4) There do not exist non-zero $\mathbb{Q}$-linear transformations $T : \mathbb{R} \to \mathbb{Q}[x]$

245. Let $M_5(\mathbb{C})$ be the complex vector space of 5×5 matrices with entries in $\mathbb{C}$. Let V be a non-zero subspace of $M_5(\mathbb{C})$ such that every non-zero $A \in V$ is invertible. Which among the following are possible values for the dimension of V?
(1) 1 (2) 2
(3) 3 (4) 5

246. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear map with four distinct eigenvalues and satisfying $T^4 - 15T^2 + 10T + 24I = 0$. Which of the following statements are necessarily true?
(1) There exists a non-zero vector $v_1 \in \mathbb{R}^4$ such that $Tv_1 = 2v_1$
(2) There exists a non-zero vector $v_2 \in \mathbb{R}^4$ such that $Tv_2 = v_2$

(3) For every non-zero vector $v \in \mathbb{R}^4$, the set $\{2v, 3Tv\}$ is linearly independent
(4) T is a one-one function

247. Let $q_1(x_1, x_2)$ and $q_2(y_1, y_2)$ be real quadratic forms such that there exist $(u_1, u_2), (v_1, v_2) \in \mathbb{R}^2$ such that $q_1(u_1, u_2) = 1 = q_2(v_1, v_2)$. Define $q(x_1, x_2, y_1, y_2) = q_1(x_1, x_2) - q_2(y_1, y_2)$. Which of the following statements are necessarily true?
(1) q is a quadratic form in x_1, x_2, y_1, y_2
(2) There exists $(t_1, t_2) \in \mathbb{R}^2$ such that $q_1(t_1, t_2) = 5$
(3) There does not exist $(s_1, s_2) \in \mathbb{R}^2$ such that $q_2(s_1, s_2) = -5$
(4) Given $\alpha \in \mathbb{R}$, there exists a vector $w \in \mathbb{R}^4$ such that $q(w) = \alpha$

248. Consider the real vector space $V = \mathbb{R}[x]$ equipped with an inner product. Let W be the subspace of V consisting of polynomials of degree at most 2. Let $W^\perp$ denote the orthogonal complement of W in V. Which of the followig statements are true?
(1) There exists a polynomial $p(x) \in W$ such that $x^4 - p(x) \in W^\perp$
(2) $W^\perp = \{0\}$
(3) W and $W^\perp$ have the same dimension over $\mathbb{R}$
(4) $W^\perp$ is an infinite dimensional vector space over $\mathbb{R}$

249. Let V be the subspace spanned by the vectors
$v_1 = (1, 0, 2, 3, 1)$, $v_2 = (0, 0, 1, 3, 5)$, $v_3 = (0, 0, 0, 0, 1)$
in the real vector space $\mathbb{R}^5$. Which of the following vectors are in V?
(1) (1, 1, 1, 1, 1) (2) (0, 0, 1, 2, 4)
(3) (1, 0, 1, 0, 1) (4) (1, 0, 1, 0, 2)

250. Let A be a 4×4 real matrix whose minimal polynomial is $x^2 + x + 1$ and let $B = A + I_4$. Which of the following statements are necessarily true?
(1) The minimal polynomial of B is $x^2 + x + 1$
(2) The minimal polynomial of B is $x^2 - x + 1$
(3) $B^3 = I_4$
(4) $B^3 + I_4 = 0$

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PART – B

251. Let V be the $\mathbb{R}$-vector space of 5×5 of real matrices. Let $S=\{AB-BA \mid A, B \in V\}$ and W denote the subspace of V spanned by S. Let $T : V \to \mathbb{R}$ be the linear transformation mapping a matrix A to its trace. Which of the following statements is true?
(1) $W = \ker(T)$
(2) $W \subsetneq \ker(T)$
(3) $W \cap \ker(T) \subsetneq W$
(4) $W \cap \ker(T) \subsetneq \ker(T)$

252. For a variable x, consider the $\mathbb{R}$ -vector space $V = \{a_0 + a_1x + a_2x^2 \mid a_1, a_2, a_3 \in \mathbb{R}\}$. Let $T:V \to V$ be the linear transformation defined by
$T(f) = f + \frac{df}{dx}$, where $\frac{df}{dx}$ denotes the derivative of f with respect to x. Which of the following statements is true?
(1) $(T^3 - 3T^2 + 3T)^{2025}(x) = x$
(2) $(T^3 - 3T^2 + 3T)^{2025}(x) = x + 1$
(3) $(T^3 - 3T^2 + 3T)^{2025}(x) = 2025!x$
(4) $(T^3 - 3T^2 + 3T)^{2025}(x) = 2025!x + 1$

253. Consider the bilinear form $B:\mathbb{R}^4 \times \mathbb{R}^4 \to \mathbb{R}$ defined by $B(x, y) = x_1y_3 + x_2y_4 - x_3y_1 - x_4y_2$, where $x=(x_1, x_2, x_3, x_4)$ and $y = (y_1, y_2, y_3, y_4)$ in $\mathbb{R}^4$. Let A denote the matrix of B with respect to the standard ordered basis of $\mathbb{R}^4$. Which of the following statements is true?
(1) Det A=0
(2) Det A =-1
(3) $B(x, x) \neq 0$ for all nonzero $x \in \mathbb{R}^4$
(4) If $x \in \mathbb{R}^4$, is nonzero, then there exists $y \in \mathbb{R}^4$ such that $B(x, y) \neq 0$

254. Let $v = (a,b,c) \in \mathbb{R}^3$ be a nonzero vector that lies in the orthogonal complement (with respect to the standard inner product) of the row-space of the matrix .

$$A=\begin{pmatrix}2 & 2 & 7\\ 3 & 1 & 4\end{pmatrix}$$

If a,b, c are all integers, then what is the smallest possible value of |a+b+c|?

(1)5 (2)10
(3)15 (4)20

255. Let U denote the span of $\{e^t, e^{2t}, e^{3t}\}$ in the real vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Consider the $\mathbb{R}$-vector spaces $V=\{f:U\to\mathbb{R}$|f is an $\mathbb{R}$-linear transformation}
$W=\{f\in V \mid f(e^{3t})=0\}$.Which of the following statements is true ?

(1) Both V and W are infinite-dimensional
(2) dim V=3 and dim W=1
(3) dim V=3 and dim W=2
(4) V is infinite-dimensional and dim W=0

256. Let $A=\begin{bmatrix}0 & a & 0\\ 0 & 0 & b\\ c & 0 & 0\end{bmatrix}$ where a, b, c are real numbers with abc=1. If B=A+A^2+A^3, then which of the following statements is true?

(1) det B=1 (2) det A=0
(3) rank(B)=2 (4) rank(B^2)=1

PART-C

257. For every integer $n\geq 2$, consider a $\mathbb{C}$-linear transformation T: $\mathbb{C}^n\to\mathbb{C}^n$. Let V be a subspace of $\mathbb{C}^n$ such that T(V)$\subseteq$ V. Which of the following statements are necessarily true?

(1) There exists a subspace W of $\mathbb{C}^n$ such that $\mathbb{C}^n$=V+M and $V\cap W=\{0\}$.
(2) There exists a subspace W of $\mathbb{C}^n$ such that T(W)$\subseteq$ W, $\mathbb{C}^n$=V+W and $V\cap W=\{0\}$.
(3) Suppose that there exists a positive integer k such that T^k is the identity map. Then there exists a subspace W of $\mathbb{C}^n$ such that T(W) $\subseteq$W, $\mathbb{C}^n$=V+W and $V\cap W=\{0\}$.
(4) Suppose that there exists a subspace W of $\mathbb{C}^n$ such that T(W)$\subseteq$ W, $\mathbb{C}^n$=V+W and $V\cap W=\{0\}$.Then there exist a positive integer k such that T^k is the identity map.

258. Let $A=\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}$ $and\ B=\begin{pmatrix}0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$.

Which of the following statements are true?

(1) Both A and B are diagonalizable over $\mathbb{R}$
(2) A is diagonalizable over $\mathbb{C}$ but not over $\mathbb{R}$
(3) Neither A nor B is diagonalizable over $\mathbb{R}$, but both A and B are diagonalizable over $\mathbb{C}$.
(4) Neither A nor B is diagonalizable over $\mathbb{C}$.

259. For a variable x, consider the $\mathbb{Q}$-vector space
$V=\{ax^3+bx^2+cx+d \mid a,b,c,d\in\mathbb{Q}\}$. Further , let $A=\{f:V\to\mathbb{Q}$ | f is a $\mathbb{Q}$-linear transformation} and
B=$\{f\in A \mid f(1)=0\}$.Which of the following statements are true?

(1) If $f\in B$, then dim ker f=3
(2) dim B=3
(3) dim A=4
(4) If $f\in A$, then the image of f is a one-dimensional $\mathbb{Q}$-vector space.

260. For a 4×4 positive definite real symmetric matrix A and real numbers a, b, c, d consider the 5×5 matrix

$$B=\left(\begin{array}{c|cccc}0 & a & b & c & d\\ \hline a & & & & \\ b & & A & & \\ c & & & & \\ d & & & & \end{array}\right)$$

Which of the following statements are necessarily true?

(1) det(B)>0 for every nonzero (a,b,c,d)$\in\mathbb{R}^4$
(2) det (B)>0 for infinitely many (a,b,c,d) $\in\mathbb{R}^4$.

(3) det(B) ≤ 0 for every (a,b,c,d) $\in \mathbb{R}^4$.
(4) det(B) $\leq$ 0 for infinitely many (a,b,c,d) $\in \mathbb{R}^4$.

261. Let $M_2(\mathbb{R})$ denote the $\mathbb{R}$-Vector space of 2×2 matrices with real entries. Let $A=\begin{pmatrix}1&2\\0&3\end{pmatrix}$ and $B=\begin{pmatrix}-1&0\\1&5\end{pmatrix}$. Define a linear transformation T:$M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by T(X)= AXB^t, where B^t denotes the transpose of the matrix B. Which of the following statements are true?
(1) det(T)=225
(2) det(T)= -225
(3) Trace(T)=16
(4) Trace(T)= -16

262. For a variable x, consider the real vector space $V=\{ax^3+bx^2+cx+d \mid a,b,c,d\in \mathbb{R}\}$. Let $D:V\to V$ be the linear transformation where D(f) is the derivative of f with respect to x, and $M:V\to V$ be the linear transformation M(f)=xD(f). Which of the following statements are true?
(1) $DM \neq MD$.
(2) D+M is invertible.
(3) DM is invertible
(4) rank(DM) =rank (MD).

263. Let V be the $\mathbb{R}$-vector space of real valued continuous functions on the $[0,\pi]$ with the inner product given by $\langle f,g\rangle=\int_0^\pi f(x)g(x)dx$. Let $S=\{\sin(x),\cos(x),\sin^2(x),\cos^2(x)\}$ and W be the subspace of V generated by S which of the following statements are true?
(1) S is a basis of W.
(2) S is an orthonormal basis of W.
(3) There exist $f,g\in S$ such that $\langle f,g\rangle=0.$
(4) S contains an orthonormal basis of W.

264. Let A, B be 2×2 matrices with real entries, and M=AB-BA. Let I_2 denote 2×2 identity matrix. Which of the following statements are necessarily true?
(1) If A and B are upper triangular, then M is diagonalizable over $\mathbb{R}$
(2) If A and B are diagonalizable over $\mathbb{R}$ then M is diagonalizable over $\mathbb{R}$
(3) If A and B are diagonalizable over $\mathbb{R}$, then there exists $\lambda\in\mathbb{R}$ such that M= λI_2.
(4) There exists $\lambda\in\mathbb{R}$ such that M^2= λI_2.

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PART - B

265. Which of the following polynomials is the characteristic polynoial of a real 2 × 2 matrix A such that trace (A) = 7 and trace (A^2) = 29?
(1) $t^2 + 7t + 10$ (2) $t^2 - 7t + 29$
(3) $t^2 - 7t - 10$ (4) $t^2 - 7t + 10$

266. Let C[0, π] be the real vector space of real-valued continuous functions on the closed interval [0, π]. For positive integers n, define $f_n \in$ C[0, π] by

$$f_n(x)=\begin{cases}\dfrac{\sin(nx)}{\sin x} & if\ x\in(0,\pi),\\ n & if\ x=0,\\ (-1)^{n-1}n & if\ x=\pi.\end{cases}$$

Let V be the real subspace of C[0, π] spanned by {f_1, f_2, f_3}. Consider the inner product on V given by

$$\langle f,g\rangle=\frac{2}{\pi}\int_0^\pi f(x)g(x)\sin^2 x\,dx.$$

Which of the following statements is true?
(1) $f_4 \in$ V.
(2) {f_1, f_2, f_3} is an orthonormal basis of V.
(3) The dimension of V is 2.
(4) {f_1, f_2, f_3} is an orthogonal set but not orthonormal.

267. Let V = {$ax^3 + bx^2 + cx$ | a, b, c $\in\mathbb{R}$}. For f $\in$ V, define

$$Q(f)=\int_{-1}^{1}(f'(t))^2\,dt,$$

where f′ denotes the derivative of f. Which of the following statements is False?
(1) Q is a positive definite quadratic form on V.
(2) Q takes every positive real value.
(3) Q(x) = 2.
(4) For all f, g $\in$ V, Q(f + g) = Q(f) + Q(g).

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268. Let X be the $\mathbb{R}$-vector space of all twice differentiable real valued functions on [0, 1]. Consider the linear map $\phi : X \to \mathbb{R}^3$ defined by $\phi(f) = (f(1), f'(1), f''(1))$. Which of the following statements is true?
(1) The dimension of X/ker ϕ is 3.
(2) ker ϕ is finite dimensional.
(3) The dimension of X/ker ϕ is 1.
(4) X is finite dimensional.

269. Consider the real matrix $A = \begin{pmatrix} 29 & 0 & 55 & 17 \\ 1 & 28 & 46 & 26 \\ 17 & 13 & 33 & 38 \\ 21 & 67 & 0 & 13 \end{pmatrix}$. What is the largest real eigenvalue of A?
(1) 101 (2) 67
(3) 103 (4) 113

PART- C

270. Let $(f_n)_{n\geq 1}$ be a sequence of real-valued functions on $\mathbb{R}$. Which of the following statements are true?
1. If each f_n is uniformly continuous and $(f_n)_{n\geq 1}$ converges to f uniformly, then f is uniformly continuous.
2. If each f_n is bounded and $(f_n)_{n\geq 1}$ converges to pointwise, then f is bounded.
3. If each f_n is bounded and continuous, $(f_n)_{n\geq 1}$ converges pointwise to a bounded and continuous function f, then the convergence is uniform.
4. If each f_n is differentiable and $(f_n)_{n\geq 1}$ converges to f uniformly, then f is differentiable.

271. Let V be a finite dimensional complex inner product space. For a linear map T: $V \to V$, let T* denote its adjoint. Which of the following statements are true?
1. If trace of TT* is zero, then T=0.
2. Let $v \in V$ be such that T*T(v)=0. Then T(v) =0.
3. Suppose T = T* and N >1 be an integer. Let $v \in V$ be such that $T^{2^N}(v) = 0$. Then T(v) =0.
4. Suppose T T* =T*T and N >1 be an integer. Let $v \in V$ be such that $T^N(v) = 0$. Then T(v) =0.

272. Let S, T: $\mathbb{C}^2 \to \mathbb{C}^2$ be $\mathbb{C}$–linear transformations and I denote the identity transformation on $\mathbb{C}^2$. Which of the following statements are necessarily true?
1. $(ST- TS)^2 = \lambda I$ for some $\lambda \in \mathbb{C}$.
2. The characteristic polynomial of $(ST-TS)^2$ is $(x-\lambda)^2$ for some $\lambda \in \mathbb{C}$.
3. If ST- TS has only one eigenvalue, then ST- TS = λI for some $\lambda \in \mathbb{C}$.
4. If ST- TS has only one eigenvalue, then $(ST-TS)^2$ is the zero transformation.

273. Let T: $\mathbb{C}^7 \to \mathbb{C}^7$ be a $\mathbb{C}$ - linear operator with eigenvalues 2, 3 and 5. Consider the subspace W:= {v $\in \mathbb{C}^7$: $(T-5I)^k$ v =0 for some integer k >0} of $\mathbb{C}^7$. Suppose that $(T-2I)^2(T-3I)^2(T-5I)^2 = 0$. Which of the following statements are necessarily true?
1. T has at least four linearly independent eigenvectors.
2. $\dim W \geq 2$.
3. $\ker((T-2I)^{2025}) = \ker((T-2I)^{2026})$
4. (T-2I) (T-3I) is nilpotent operator.

274. Which of the following matrices are similar over $\mathbb{R}$ to the matrix
$$A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
1. $\begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ 2. $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
3. $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ 4. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

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275. Let A, B be distinct 2×2 real matrices. Which of the following statements are true?
1. If A is invertible, then AB and BA have the same minimal polynomial.
2. If 0 is an eigenvalue of A, then 0 is an eigenvalue of AB.
3. If 0 is the only eigenvalue of A and of B, then 0 is the only eigenvalue of AB.
4. If AB and BA have the same minimal polynomial, then either A or B is invertible.

276. Let $M_2(\mathbb{R})$ denote the space of real 2×2 matrices. Let S be the vector subspace of $M_2(\mathbb{R})$ comprising of all symmetric matrices. Let F: $M_2(\mathbb{R}) \to S$ be the map defined by $F(X) = XX^T$. Let DF_A: $M_2(\mathbb{R}) \to S$ be the derivative of F at $A\in M_2(\mathbb{R})$. Which of the following statements are true?
1. If $AA^T = I$, then DF_A: $M_2(\mathbb{R}) \to S$ is surjective.
2. If $AA^T = I$, then DF_A: $M_2(\mathbb{R}) \to S$ need not be surjective.
3. If A is invertible, then DF_A: $M_2(\mathbb{R}) \to S$ is surjective.
4. If A is not invertible, then DF_A: $M_2(\mathbb{R}) \to S$ is surjective.

277. Let V be the $\mathbb{R}$ –vector space of all polynomials with real coefficients. Let $f(x) = x^2+x+1$.
Which of the following subsets of V are linearly independent?
1. $\{f'(x), f(x)-f(x-1), 1\}$
2. $\{f(x+1), -f(x), f(x)-f(x-1), 1\}$
3. $\{f(x), f'(x), 1\}$
4. $\{f(x+1), f(x-1), f(x)\}$

278. Let B (v, w) be a non-degenerate symmetric bilinear form on $\mathbb{R}^2$ and let q(v) = B(v, v) be the corresponding quadratic from. Suppose there exist vectors v, w $\in \mathbb{R}^2$ such that B(v, v) =0 and B(v, w) $\neq$ 0. Which of the following statements are necessarily true?
1. B(w, w) =0
2. There exist an $\alpha \in \mathbb{R}$ such that q(α v+w) =0
3. There are infinitely many $\alpha \in \mathbb{R}$ such that q(α v+w) =0
4. q is equivalent to the quadratic form $Q(x,y) = x^2 - y^2$ for all (x, y) $\in \mathbb{R}^2$.

279. Consider the field $\mathbb{F}_3$ consisting of 3 elements. Let V be an $\mathbb{F}_3$–vector space of dimension 3 and W an $\mathbb{F}_3$–vector space of dimension 2. Which of the following statements are true?
1. The number of two dimensional subspaces of V is 13.
2. The number of surjective linear transformation from V to W is 624.
3. The number of one dimensional subspaces of V is 13.
4. The number of linear transformation from V to W is 3^6.

LINEAR ALGEBRA

PREVIOUS YEAR PAPERS

ANSWERS

1. (3)	2. (4)	3. (3)
4. (2)	5. (1,2,3,4)	6. (1,2,3)
7. (1,2,3,4)	8. (1,3,4)	9. (1,3)
10. (1,2,3)	11. (1,3,4)	12. (2)
13. (1)	14. (2)	15. (1)
16. (3)	17. (4)	18. (1, 3)
19. (1, 2, 3)	20. (2, 3, 4)	21. (1,2,3,4)
22. (3, 4)	23. (1, 2)	24. (1, 3, 4)
25. (2, 3) 1	26. (1, 3)	27. (2, 3)
28. (1, 2)	29. (1, 3, 4)	30. (1)
31. (4)	32. (2)	33. (1, 2)
34. (2)	35. (3)	36. (3)
37. (1, 2)	38. (1)	39. (1, 2)
40. (1, 3, 4)	41. (1, 2)	42. (2, 3)
43. (1, 3, 4)	44. (1, 2)	45. (1, 2)
46. (3, 4)	47. (3, 4)	48. (4)
49. (2)	50. (1)	51. (4)
52. (4)	53. (4)	54. (1, 3)
55. (2, 4)	56. (1, 3)	57. (1, 3, 4)
58. (1,2,3,4)	59. (1,2,3,4)	60. (1)
61. (2)	62. (2)	63. (1)
64. (3)	65. (4)	66. (1, 2)
67. (3, 4)	68. (2, 3)	69. (2, 3, 4)
70. (3)	71. (2)	72. (1,2,3,4)
73. (1, 2)	74. (1, 2)	75. (1)
76. (1)	77. (3)	78. (2)
79. (4)	80. (3)	81. (1)
82. (1)	83. (1, 4)	84. (1,2,3,4)
85. (3, 4)	86. (1, 3)	87. (1)
88. (2, 3)	89. (2, 3)	90. (2)
91. (1)	92. (2)	93. (2)
94. (4)	95. (3)	96. (2, 4)
97. (2, 3)	98. (2, 4)	99. (1, 3, 4)
100. (1, 2, 4)	101. (1, 3)	102. (1, 4)
103. (1, 4)	104. (1)	105. (4)
106. (4)	107. (4)	108. (3)
109. (1, 4)	110. (3, 4)	111. (2, 3)
112. (1, 4)	113. (1,2,3)	114. (1,2,3)
115. (3)	116. (4)	117. (3)
118. (4)	119. (3)	120. (4)
121. (2, 4)	122. (4)	123. (3, 4)
124. (1, 4)	125. (4)	126. (1, 4)
127. (2, 3)	128. (3, 4)	129. (1)
130. (3)	131. (3)	132. (4)
133. (2)	134. (2)	135. (1, 4)
136. (1, 3)	137. (1, 4)	138. (3, 4)
139. (4)	140. (1, 4)	141. (4)
142. (2)	143. (2)	144. (3)
145. (1)	146. (4)	147. (3)
148. (3)	149. (2, 3)	150. (4,3)
151. (1, 3)	152. (1)	153. (4)

154. (1, 2, 4)	155. (2, 3)	156. (1,3,4)
157. (4)	158. (3)	159. (3)
160. (4)	161. (3)	162. (3)
163. (2, 3)	164. (3, 4)	165. (2, 3)
166. (2, 4)	167. (2)	168. (1, 2)
169. (3)	170. (3)	171. (2)
172. (4)	173. (4)	174. (2)
175. (2, 3)	176. (1,2)	177. (2, 4)
178. (1, 3)	179. (1, 2)	180. (1,2,3,4)
181. (1,2,4)	182. (2, 3)	183. (3)
184. (3)	185. (4)	186. (1)
187. (2)	188. (1)	189. (1,2,3,4)
190. (1,3)	191. (2,4)	192. (1,3)
193. (4)	194. (4)	195. (2,3,4)
196. (2)	197. (3)	198. (4)
199. (4)	200. (1)	201. (4)
202. (1)	203. (3)	204. (4)
205. (1,3,4)	206. (1,2)	207.
208. (2,4)	209. (4)	210. (3)
211. (2)	212. (4)	213. (4)
214. (2)	215. (2, 4)	216. (2)
217. (2,3)	218. (1,2,3)	219. (1,2)
220. (1,3)	221. (1,2,3,4)	222. (*)
223. (2)	224. (3)	225. (3)
226. (4)	227. (2)	228. (4)
229. (3,4)	230. (1)	231. (1,4)
232. (1,3,4)	233. (1,3)	234. (2,3)
235. (1,2,3,4)	236. (2,3)	237. (3)
238. (3)	239. (1)	240. (2)
241. (4)	242. (2)	243.
244. (2)	245. (1)	246. (1,4)
247. (1,2,4)	248.	249. (3,4)
250. (2,4)	251. (1)	252. (1)
253. (4)	254. (2)	255. (3)
256. (4)	257. (1,3)	258. (2,3)
259. (2,3)	260. (3,4)	261. (1,3)
262. (1)	263. (1,3)	264. (4)
265. (4)	266. (2)	267. (4)
268. (1)	269. (1)	270. (1)
271. (1,2,3,4)	272. (1,2,4)	273. (1,3)
274. (1,3)	275. (1,2)	276. (1,3)
277. (3,4)	278. (2,4)	279. (1,2,3,4)